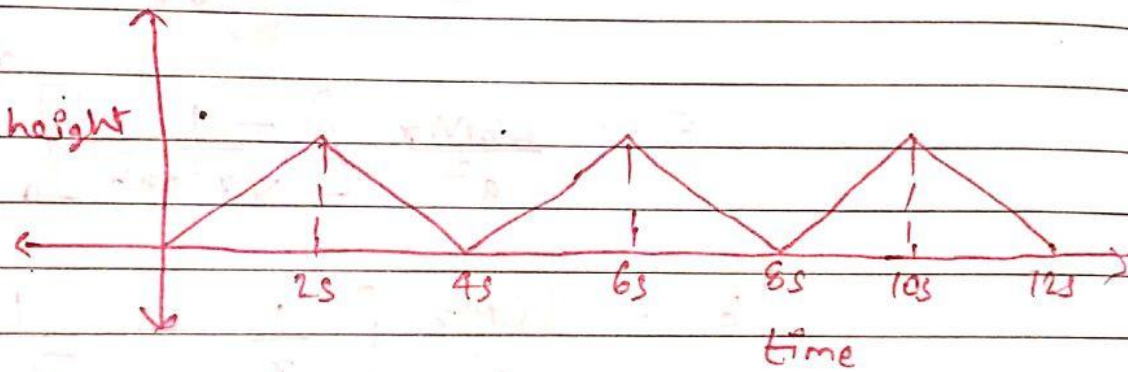
# S. H. M

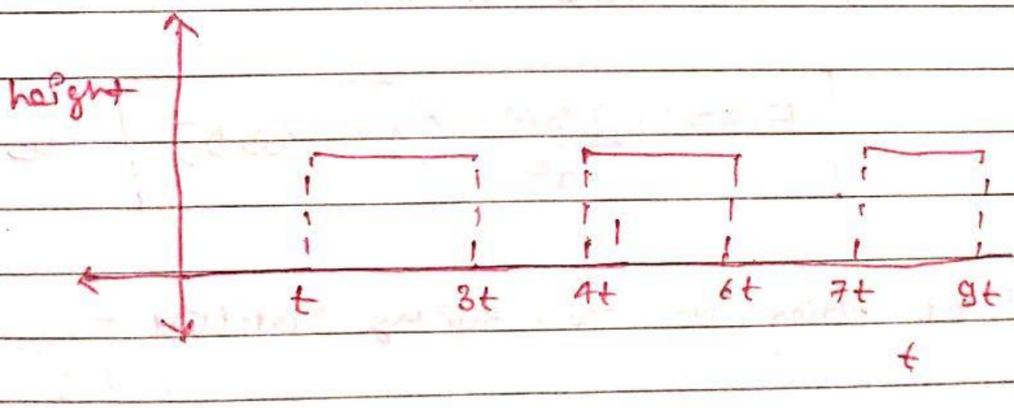
# SHM is not a wave

- Periodic Motion : (Fixed) Regular Interval Repeat
  Harmonic

① insect climbing a wall & falling down



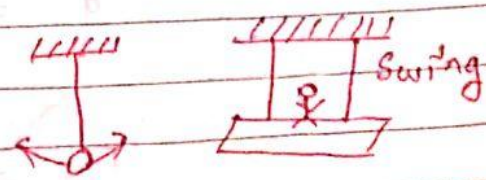

② Bacha → Steps → Climbs up & down




③ U.C.M

- Oscillatory : to & fro periodic

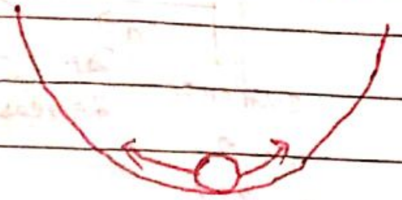
- All oscillatory motion are periodic (99%).



• Oscillatory ⟶ equation : $F = -Kx^n$

$\{n = 1, 3, 5, 7\}$

$x^n$ ↓ measured from mean position

$x = 0$

Mean position (Stable Equilibrium position)

Force ↓ Mean position

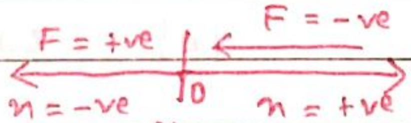
Pe लाना चाहती है।

"Yess. All oscillatory motions are periodic ~~tab~~ except in which Energy is lost."

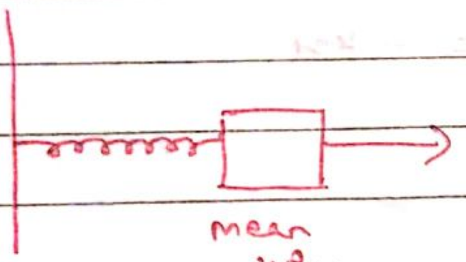
• SHM ⟶ Special case / simplest case of Oscillation

↓ {St. line}

$$\boxed{F = -Kx^1}$$




$x$ → measured from mean position



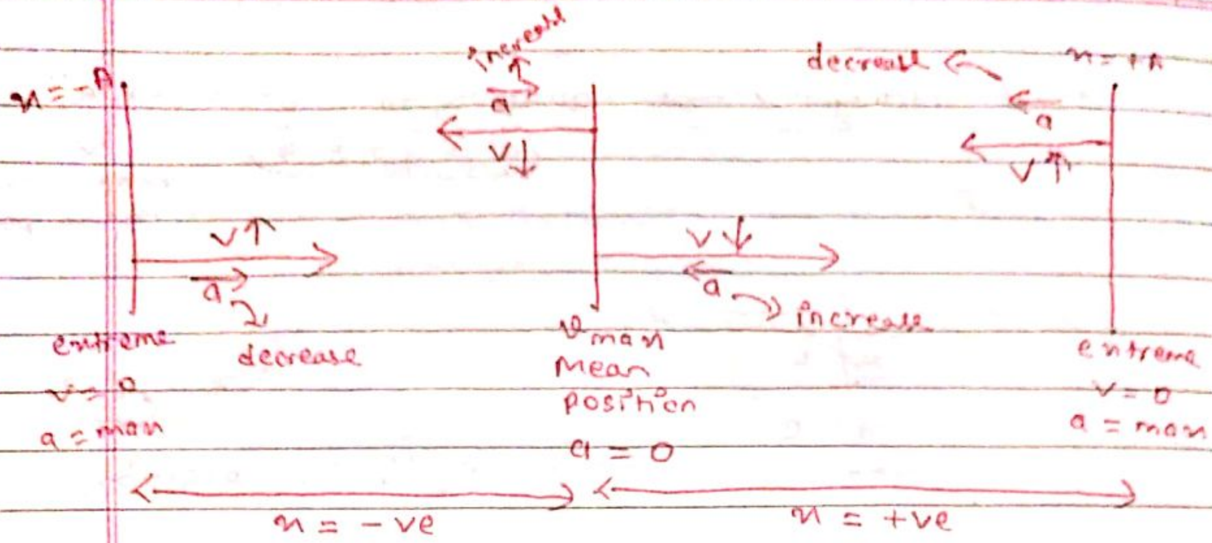

⟶ $a_{max} = \omega A$

extreme $x = \pm A$

⟶ $v = 0$ extreme

$V = \pm \omega \sqrt{A^2 - x^2}$ ⟶ $V_{max} = \pm A\omega$

mean




$$F = -kx$$

$$a = \frac{F}{m} = -\frac{kx}{m}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

$$\boxed{a = -\omega^2 x}$$

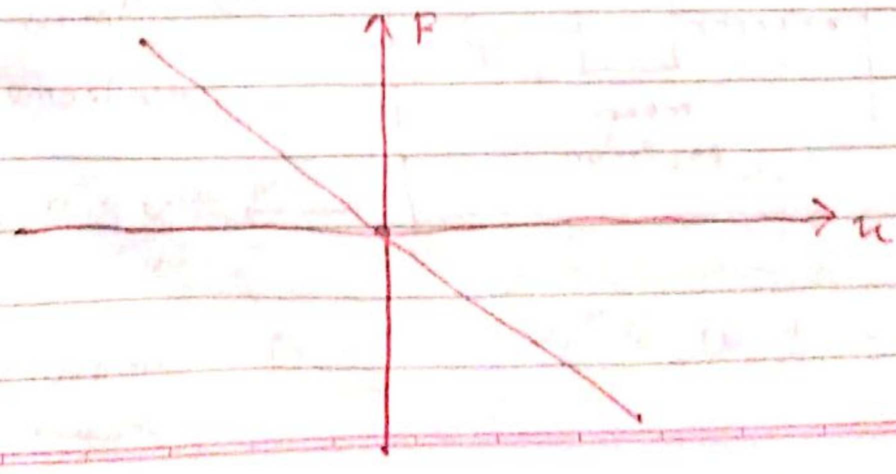
$a \rightarrow max$, when $x = +A$ or $x = -A$

entreme position

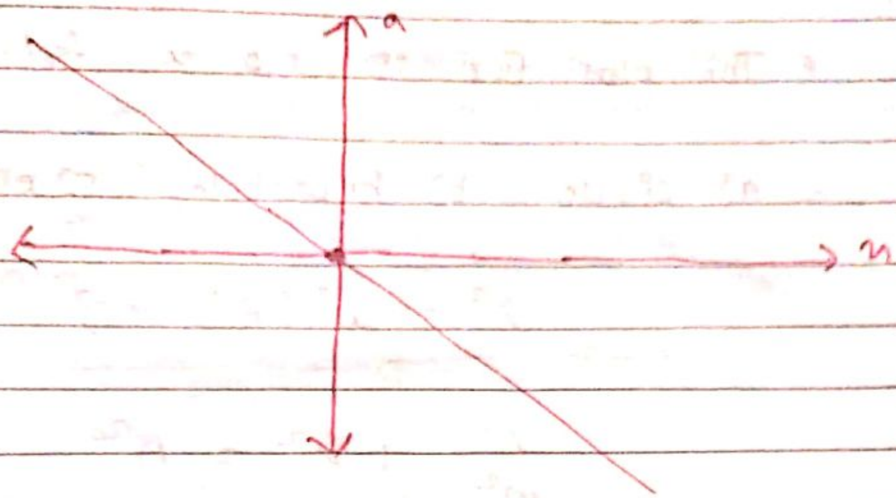
$$|a| = \omega^2 A$$

$a \rightarrow 0$, $x = 0$ (mean position)

- F v/s x $\quad F = -kx$



• a v/s x $\quad a = -\omega^2 x$




• v v/s x

$$a = -\omega^2 x$$

$$\frac{dv}{dt} \times \frac{dx}{dx} = -\omega^2 x$$

$$\frac{v\,dv}{dx} = -\omega^2 x$$

$$\int_0^v v\,dv = -\int_{x=+A}^{x} \omega^2 x\,dx$$

$$\left[\frac{v^2}{2}\right]_0^v = -\omega^2\left[\frac{x^2}{2}\right]_A^x$$

$$\frac{v^2}{2} = -\frac{\omega^2}{2}(x^2 - A^2)$$

$$\boxed{v^2 = \omega^2(A^2 - x^2)}$$

• The plot for $v$ v/s $x$ is: (in SHM)

a) circle  b) hyperbola  c) ellipse  d) parabola

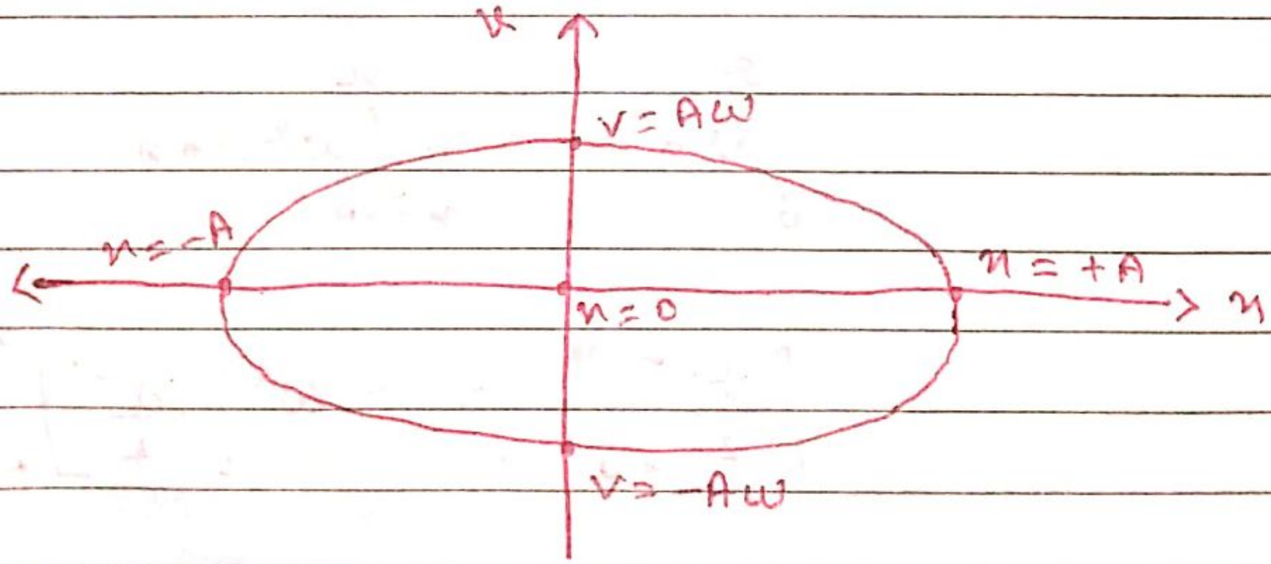
$$v^2 = \omega^2(A^2 - x^2)$$

$$\frac{v^2}{\omega^2} + x^2 = A^2$$

$$\boxed{\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1}$$

Ellipse $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right)$

$$\frac{x^2}{A^2} + \frac{v^2}{(\omega A)^2} = 1$$




$$\boxed{V_{max} = \pm A\omega}$$

Mean position

$$v = \pm \omega \sqrt{A^2 - x^2}$$

$$\frac{dx}{dt} = \pm \omega \sqrt{A^2 - x^2}$$

$$\int_0^x \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \pm \omega \, dt$$

at $t = 0$ initially particle is at mean position $x = 0$

let $x = A\sin\theta$

$$\sqrt{A^2 - x^2} = \sqrt{A^2(1 - \sin^2\theta)} = A\cos\theta$$

$$dx = A\cos\theta \, d\theta$$

$$\int_0^x \frac{A\cos\theta \, d\theta}{A\cos\theta} = \int_0^t \pm \omega \, dt$$

$$\left[\sin^{-1}\left(\frac{x}{A}\right)\right]_0^x = \pm \omega (t)_0^t$$

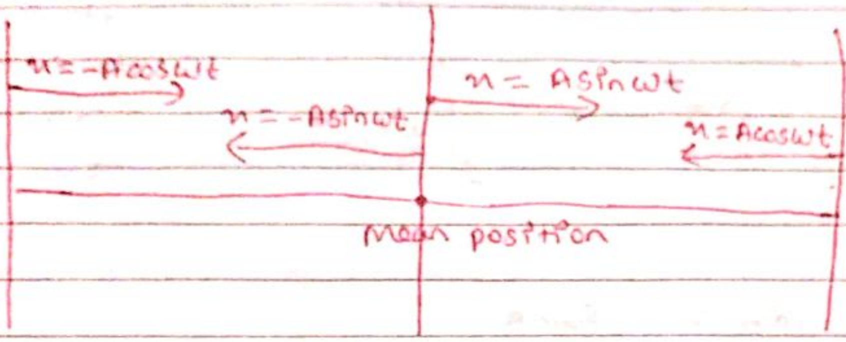
$$\sin^{-1}\frac{x}{A} = \pm \omega t$$

$$\frac{x}{A} = \sin(\pm \omega t)$$

$$\boxed{x = A\sin(\pm \omega t)}$$

Scanned by CamScanner

$$\boxed{\begin{aligned} x &= A\sin\omega t \\ x &= -A\sin\omega t \end{aligned}}$$ or $x = A\sin\omega t$




- At $t = 0$ particle is at $x = +A$ (extreme)

$$\int_{+A}^{x} \frac{dx}{\sqrt{A^2 - x^2}} = \int_{0}^{t} \pm\omega\, dt$$

$$\left[\sin^{-1}\left(\frac{x}{A}\right)\right]_{A}^{x} = \pm\omega(t)_0^t$$

$$\sin^{-1}\left(\frac{x}{A}\right) - \sin^{-1}\left(\frac{A}{A}\right) = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) - \frac{\pi}{2} = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) = \frac{\pi}{2} \pm \omega t$$

$\cancel{\sin^{-1}}(-1) = \sin\frac{3\pi}{2}$

Date________
Page________

$$x = A\sin\left(\frac{\pi}{2} \pm \omega t\right)$$

$$\boxed{x = A\cos\omega t}$$

- At $t = 0$ particle is at $x = -A$

$$\left[\sin^{-1}\left(\frac{x}{A}\right)\right]_{-A}^{x} = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) - \sin^{-1}\left(-\frac{A}{A}\right) = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) - \frac{3\pi}{2} = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) = \frac{3\pi}{2} \pm \omega t$$

$$x = A\sin\left(\frac{3\pi}{2} \pm \omega t\right)$$

$$\boxed{x = -A\cos\omega t}$$

# General eqn :-

$$\boxed{\begin{aligned} x &= A\sin(\omega t + \phi) \\ &\text{or} \\ x &= A\cos(\omega t + \phi) \end{aligned}}$$

$x = A\sin(\omega t + \phi)$

$x = A\cos(\omega t + \phi)$

Date
Page

$\phi \rightarrow$ initial phase, $(\omega t + \phi) \rightarrow$ phase

- $$x = A\sin(\omega t + \phi)$$

$$\frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

$$\boxed{v = A\omega\cos(\omega t + \phi)}$$

$$a = \frac{dv}{dt} = A\omega^2(-\sin(\omega t + \phi))$$

$$\boxed{a = -A\omega^2\sin(\omega t + \phi)}$$

$\boxed{a = -\omega^2 x}$ differential equation of SHM

$$\frac{dv}{dt} = -\omega^2 x$$

$$\frac{d\left(\frac{dx}{dt}\right)}{dt} = -\omega^2 x$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

Scanned by CamScanner

Date
Page

$$\boxed{\begin{aligned} v_{max} &= \pm A\omega \\ a_{max} &= \pm A\omega^2 \end{aligned}}$$

- Time period of 'SHM'

$$x_1 = x_2$$

$$A\sin(\omega t + \phi) = A\sin(\omega(t+T) + \phi)$$

$$\sin\theta = \sin\alpha$$

$$\theta = \alpha, \quad \theta = 2\pi + \alpha \checkmark$$

$$\theta = \pi - \alpha$$

$$v = A\omega\cos(\omega t + \phi)$$

$$v_1 = v_2$$

$$\cancel{A\omega}\cos(\omega t + \phi) = \cancel{A\omega}\cos(\omega(t+T) + \phi)$$

$$\cos\theta = \cos\alpha,$$

$$\theta = \alpha, \quad \theta = 2\pi + \alpha \checkmark$$

$$\theta = 2\pi - \alpha$$

$$\cancel{\omega t} + \cancel{\phi} + 2\pi = \cancel{\omega t} + \omega T + \cancel{\phi}$$

$$\omega T = 2\pi$$

$$\boxed{T = \frac{2\pi}{\omega}} \checkmark$$

Date
Page

Q) Write the eq^n for SHM ($x - t$) if initially particle is at origin & moving in -ve direction w.r.t mean position.

(Mean position = origin) ($A = 2cm$, $T = 4s$)

$$\Rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\omega = \frac{\cancel{2}\pi}{\cancel{4}\,2} = \frac{\pi}{2}$$

$$x = -2\sin\left(\frac{\pi}{2}t\right)$$

$$\boxed{x = -2\sin\left(\frac{\pi}{2}t\right)}$$

OR

$$x = A\sin(\omega t + \phi) \qquad v = A\omega\cos(\omega t + \phi)$$

At $t = 0$, $x = 0$

$$0 = A\sin(0 + \phi) \qquad v < 0, \; t = 0$$

$$\sin\phi = 0 \qquad A\omega\cos(0 + \phi) < 0$$

$$\phi = 0 \text{ or } \pi \qquad \cos\phi < 0$$

$$\cos 0 = 1 \quad \cos 180^\circ = -1$$

$$\boxed{\phi = \pi}$$

$$x = 2\sin(\omega t + \pi)$$

$$\boxed{x = -2\sin\left(\frac{\pi}{2}t\right)}$$

$\boxed{0 \leq \phi \leq 2\pi}$

Date ______
Page ______

Q) Write the eq^n for SHM (x - t), (v - t) & find $v_s$ at t = 2sec, 3sec if initially particle is at Right extreme of origin & moving in -ve x-direction. (A = 2cm, T = 8 sec)

$\Rightarrow$

$x = A\sin(\omega t + \phi)$

At t = 0, x = A

$\cancel{A} = \cancel{A}\sin(0 + \phi)$

$\sin\phi = 1$

$\phi = \frac{\pi}{2}$

$x = A\sin\left(\frac{\pi}{2} + \omega t\right)$

$= A\cos\omega t$

$T = \frac{2\pi}{\omega}$

$\cancel{8}\,4 = \frac{2\pi}{\omega}$

$\omega = \frac{\pi}{4}$

$x = 2 \times \cos\left(\frac{\pi}{4}t\right)$

$\boxed{x = 2\cos\left(\frac{\pi}{4}t\right)}$

$v = A\omega\cos(\omega t + \phi)$

$v = A\omega\cos\left(\frac{\pi}{2} + \omega t\right)$

$v = -A\omega\sin(\omega t)$

$= -\frac{\cancel{2} \times \pi}{\cancel{4}\,2}\sin\left(\frac{\pi}{4}t\right)$

$= -\frac{\pi}{2}\sin\left(\frac{\pi}{4}t\right)$

At t = 2sec,

$v = -\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right)$

$\boxed{v = -\frac{\pi}{2}}$

At t = 3 sec,

$v = -\frac{\pi}{2}\sin\left(\frac{3\pi}{4}\right)$

$\boxed{v = -\frac{\pi}{2\sqrt{2}}}$ ..

At t = 4sec, $\boxed{v = 0}$ ..

$t_1 < t_2$

Date
Page

Q.3) Find $v-t$ for SHM, if initially particle is at $+\frac{A}{\sqrt{2}}$ of origin & moving initially in +ve direction.

$\Rightarrow \quad x = A\sin(\omega t + \phi) \qquad v = A\omega\cos(\omega t + \phi)$

At $t=0$

$x = \frac{A}{\sqrt{2}} \qquad \frac{A}{\sqrt{2}} = A\sin\phi \qquad A\omega\cos(\omega t+\phi) > 0$

$\cos(\phi) > 0$

$\sin\phi = \frac{1}{\sqrt{2}} \qquad \phi = \frac{\pi}{4}$

$\phi = \frac{\pi}{4}$

$$\boxed{v = A\omega\cos\left(\omega t + \frac{\pi}{4}\right)}$$

# IIT 2001 :

If initially particle is at origin (SHM) & it ~~moves 'a' distance in~~ covers $0 \longrightarrow \frac{A}{2}$ in $t_1$ time & $\frac{A}{2} \longrightarrow A$ in $t_2$ time.

a) $t_2 = t_1$ b) $t_2 = t_1/2$

c) $t_2 = 2t_1$ d) $t_2 = t_1 + 2$

$\Rightarrow \quad x = A\sin\omega t$

$\frac{A}{2} = A\sin\omega t_1$

$\sin \omega t_1 = \frac{1}{2}$

$\omega t_1 = \frac{\pi}{6}$

$\boxed{t_1 = \frac{\pi}{6\omega}}$

$\boxed{t_2 = \frac{\pi}{3\omega}}$

$x = A$

$t = (t_1 + t_2)$

$A = A \sin \omega (t_1 + t_2)$

$\omega (t_1 + t_2) = \frac{\pi}{2}$

$\omega t_2 = \frac{\pi}{2} - \frac{\pi}{6}$

$\omega t_2 = \frac{\pi}{3}$

# JEE Mains 2014:

SHM $\rightarrow$ A particle starts from rest & covers 'a' distance in '$\tau$' sec & another '2a' distance in next '$\tau$' sec in the same direction. Which of the following is correct?

a) $A = 3a$ b) $A = 4a$

c) $T = 6\tau$ d) $T = 8\tau$

$\Rightarrow$

$x = A \cos \omega t$

At $T = \tau$

$x = a$

$a = A \cos \omega \tau$

At $t = 2\tau$

$x = 3a$

$3a = A \cos 2\omega\tau$

$3 \times A \cos \omega\tau = A \cos 2\omega\tau$

$3 \cos \omega\tau = \cos 2\omega\tau$

$a = \cancel{A\sin\omega t}\ A\cos\omega(t + T_0)$

$t = 0$
$v = 0$
extreme

$\Rightarrow$ At $t = T_1$
$x = A - a$

$A - a = A\cos\omega t$

Scanned by CamScanner

Date
Page

At, $t = 2T$

$x = A - 3a$

$A - 3a = A\cos\omega 2T$

$\left[\cos\omega T = \frac{A-a}{A}\right]$

$= A(2\cos^2\omega t - 1)$

$A - 3a = A\left(2\left(\frac{A-a}{A}\right)^2 - 1\right)$

$A - 3a = A\left[\frac{(A^2 + a^2 - 2Aa)2}{A^2} - 1\right]$

$= A\left[\frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}\right]$

$\cancel{A^2} - 3aA = \cancel{A^2} + 2a^2 - 4Aa$

$Aa = 2a^2$

$\boxed{A = 2a}$

$A - a = A\cos\omega t$

$\cancel{a} = 2\cancel{a}\cos\omega t$

$\cos\omega t = \frac{1}{2}$

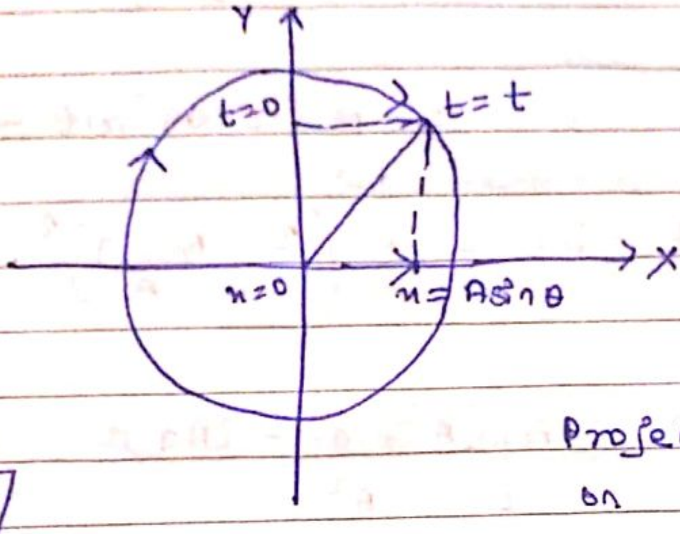
$\omega t = \frac{\pi}{3}$

$\frac{2\cancel{\pi}}{T} \times t = \frac{\cancel{\pi}}{3}$

Scanned by CamScanner

$$\boxed{T = 6\tau}$$

• SHM & Circular Motion :




Projection is doing to & fro motion

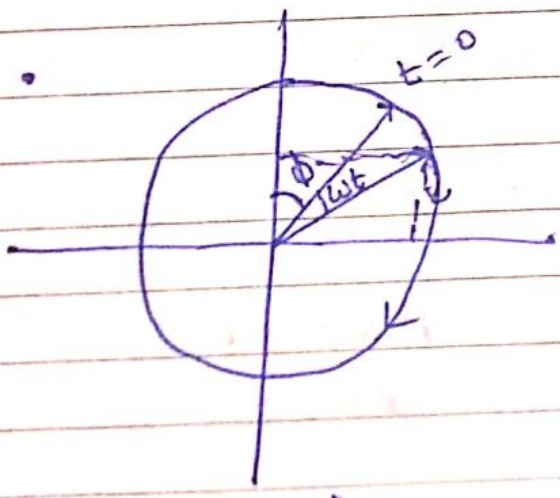
$$\boxed{\theta = \omega t}$$

Projection of particle on diameter

$$x = A\sin\theta$$

$$\boxed{x = A\sin\omega t}$$ ⟹ Yes SHM

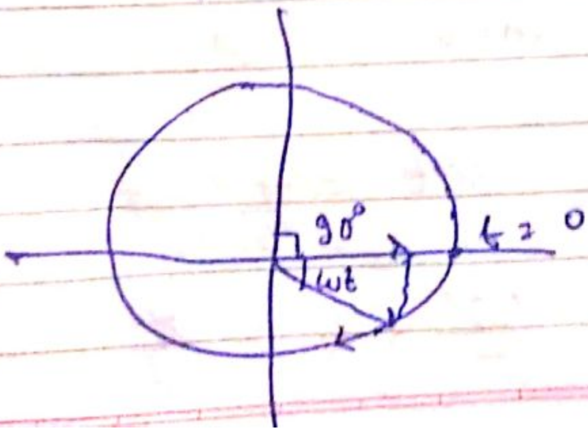
•



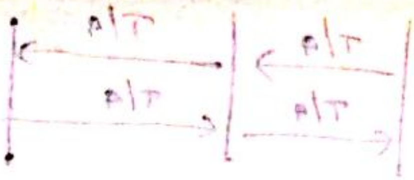

$$\boxed{x = A\sin(\omega t + \phi)}$$

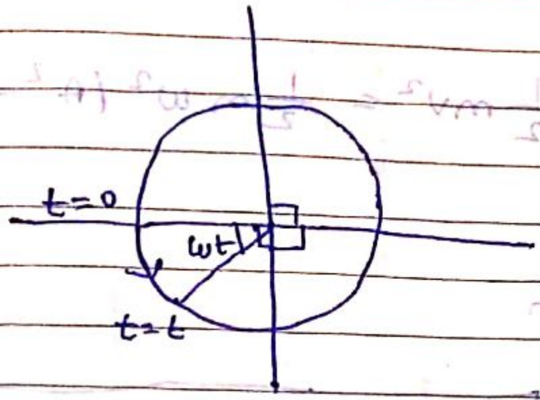
Equation for SHM

•




$$\boxed{x = A\cos\omega t}$$



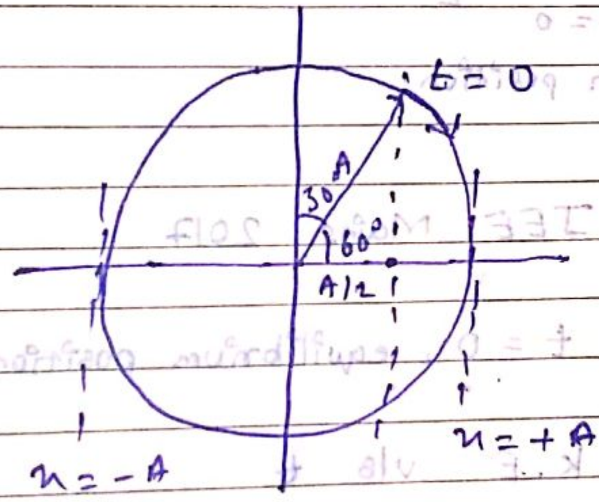

$$x = A\sin(\omega t + \phi)$$

$$x = A\sin(270 - \omega t)$$

$$\boxed{x = -A\cos\omega t}$$

Q) SHM : $t = 0$ initially $x = +\frac{A}{2}$ & initially it is moving in +ve $x$ direction. Find equation for $x$.

⇒




$$x = A\sin(\omega t + \phi)$$

$$\boxed{x = A\sin\left(\omega t + \frac{\pi}{6}\right)}$$ Ans

# SHM

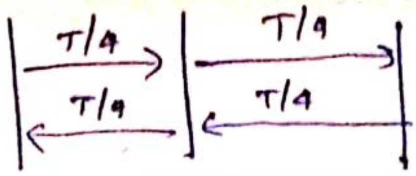
$F = -Kx$

↳ conservative → Potential Energy (variable)

Kinetic Energy (variable) + Potential Energy (variable) → Total Mechanical Energy

⇓

constant




- $K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

$\omega^2 = \frac{k}{m}$

$$\therefore \boxed{K.E = \frac{1}{2}k(A^2 - x^2)}$$

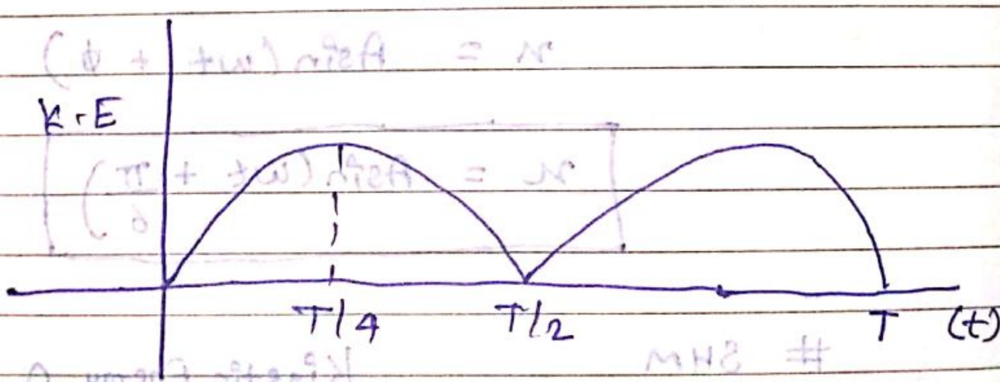
Max $= \frac{1}{2}kA^2$
$x = 0$
Mean position

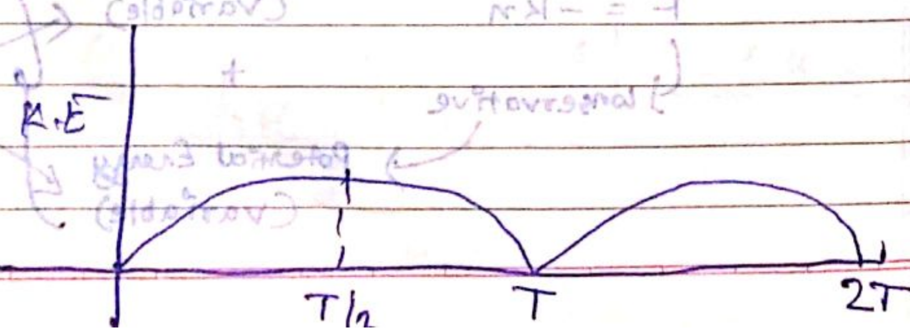
Min $= 0$
$x = \pm A$
extreme position

# JEE Mains 2017

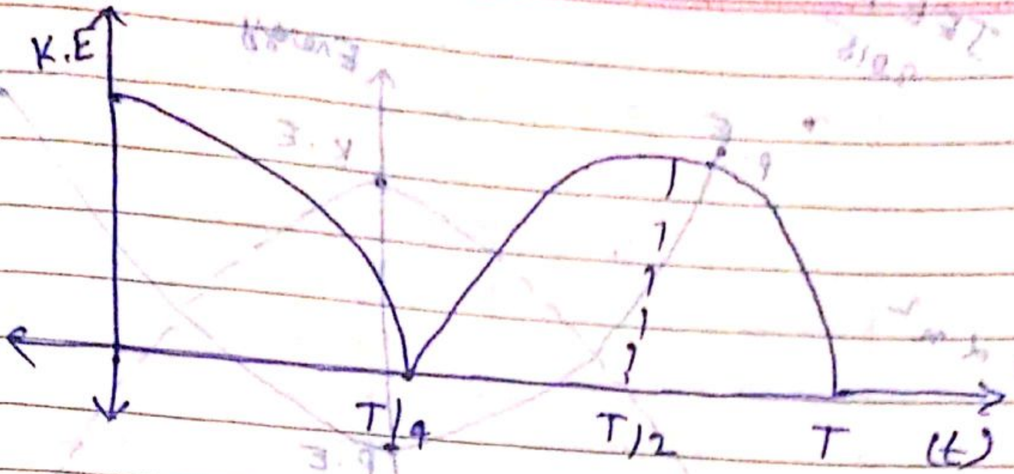
$t = 0$, equilibrium position (Mean position)

K.E v/s t

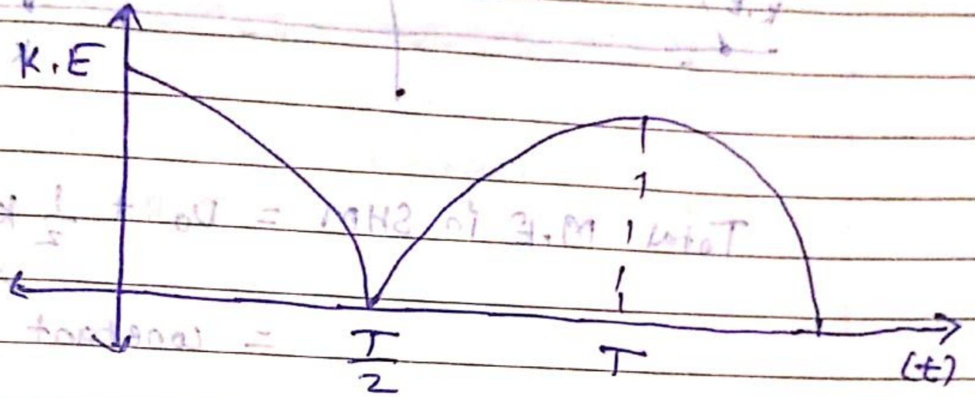
~~a)~~



~~b)~~


c)




d)




P.E $\quad \Delta U = -W_{\text{conservative force}}$

↓

$F = -kx$

conservative force

$$\int dU = -\int dW_c$$

mean position

↓

$U_0$

$$\int_{U_0}^{U} dU = -\int_{x=0}^{x=x} kx\,dx \cos 180^\circ$$

$$\boxed{U - U_0 = \frac{1}{2}kx^2}$$

$$\boxed{U = U_0 + \frac{1}{2}kx^2}$$

Min, $x = 0$, mean position, $U_{min} = U_0$

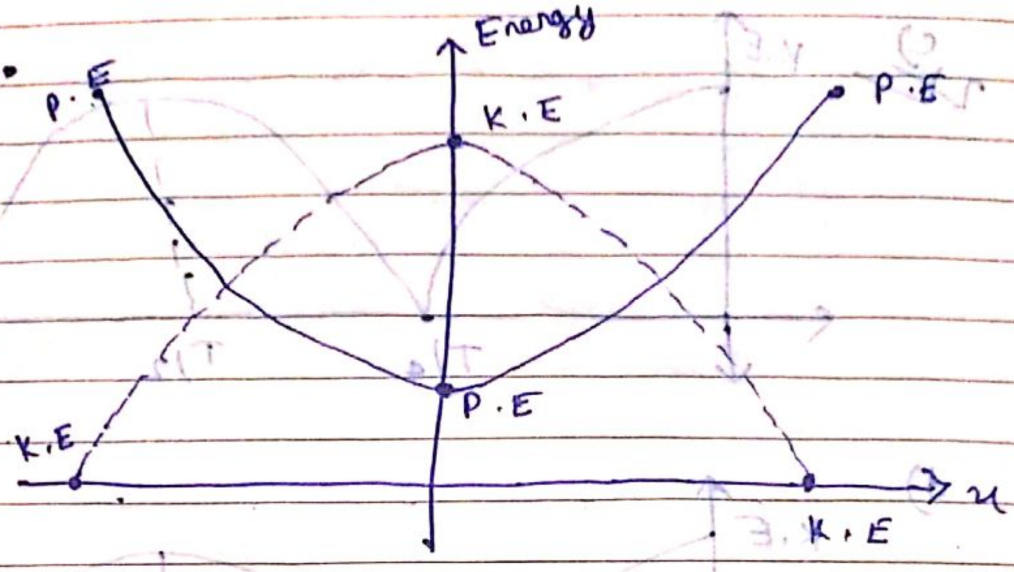
Max, $x = \pm A$, extreme, $U_{max} = U_0 + \frac{1}{2}kA^2$

$F = -kx$ $\quad \frac{-k}{m} = \omega^2$

$a = \frac{-kx}{m}$

# JEE Mains 2015

$U \propto x^2$




Total M.E in SHM $= U_0 + \frac{1}{2}kA^2$

$= \text{constant}$

Q) Assuming $U = 0$ at mean position Find Total M.E of particle undergoing SHM of mass 2 kg,

$$x = 2\sin\left(\frac{\pi}{4}t + \frac{\pi}{4}\right)$$

$A = 2$

$\omega = \frac{\pi}{4}$

$m = 2\,kg$

$k = \omega^2 m$

$= \frac{\pi^2}{16} \times 2$

$= \frac{\pi^2}{8}$

$$T.E = \frac{1}{2}KA^2$$

$$= \frac{1}{2} \times \frac{\pi^2}{8^2} \times 4)$$

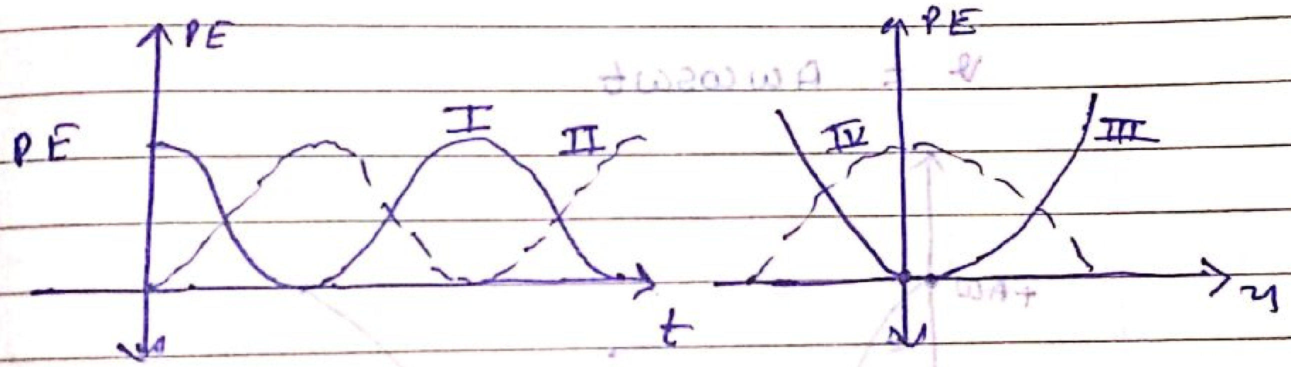
$$= \frac{\pi^2}{4}$$

# IIT 2003:

SHM, $x = A\cos\omega t$

P.E v/s $t$

P.E v/s $x$




a) I & IV  b) II & III  c) I & III  d) II & IV

- $PE = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$

- $K.E = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$

$$= \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

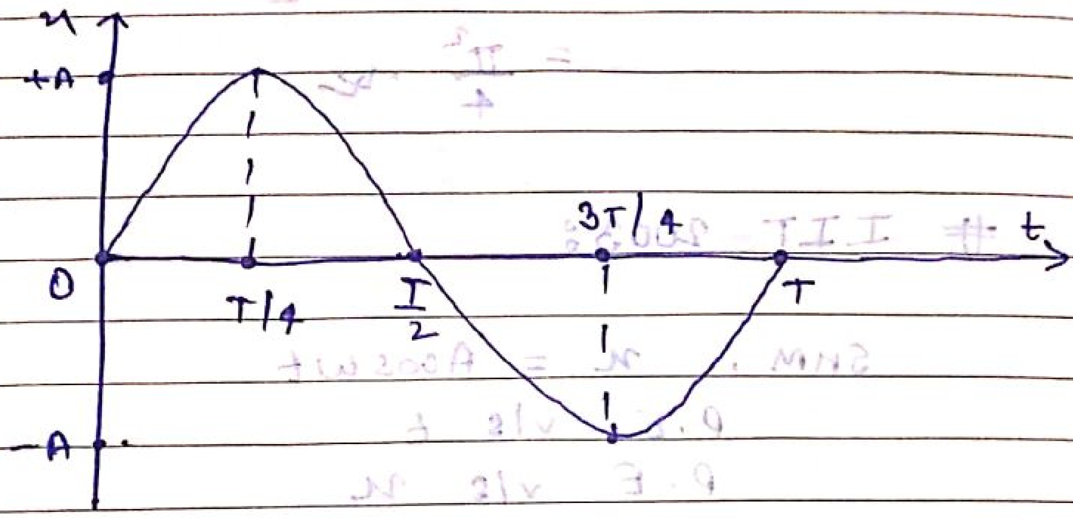
- $TE = \frac{1}{2}kA^2$

Date ____
Page ____

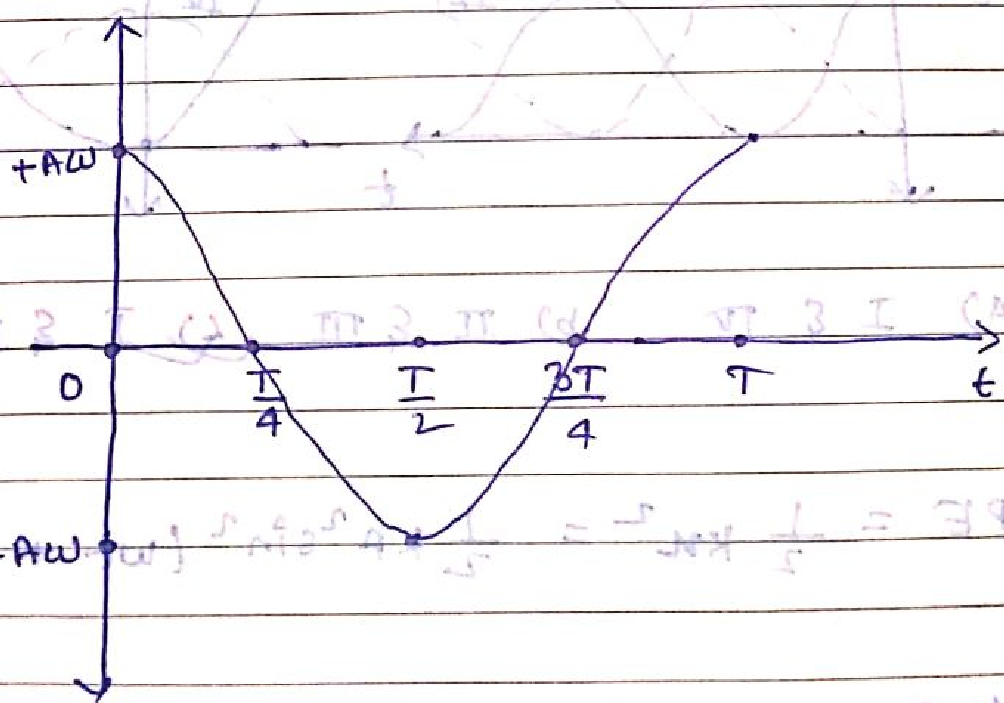
- $x = A\sin\omega t$

$x = A\sin\frac{2\pi}{T} \times t$

$t = 0$
$x = 0$
+ve direction motion



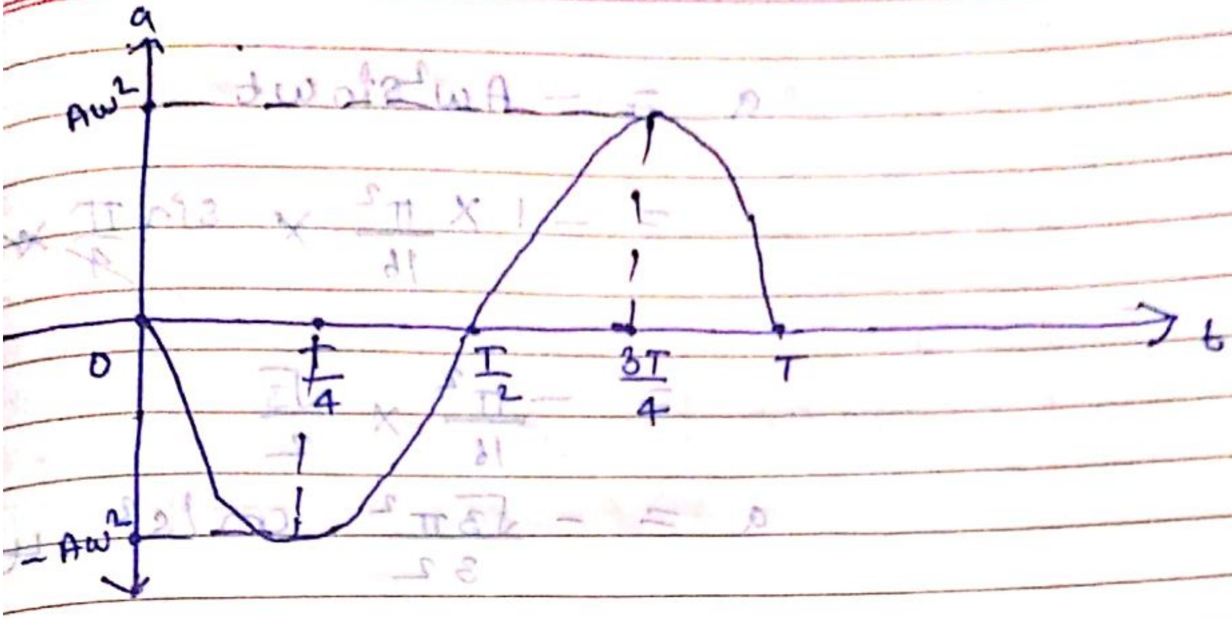

$v = A\omega\cos\omega t$



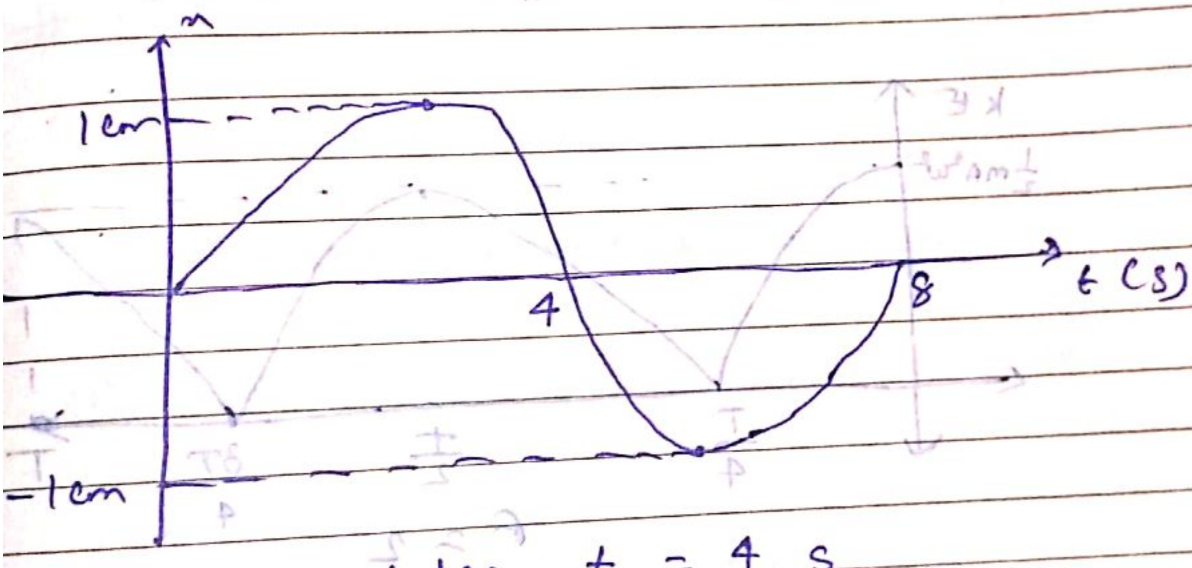

$a = -A\omega^2\sin\omega t = -A\omega^2\sin\frac{2\pi}{T}t$

$= -\omega^2 x$

Scanned by CamScanner




# IIT 2009




a when $t = \frac{4}{3}$ s

a) $\frac{\pi^2}{32}$ cm/s² b) $-\frac{\pi^2}{32}$ cm/s² c) $\frac{\sqrt{3}\pi^2}{32}$ cm/s²

d) $\left(-\frac{\sqrt{3}}{32}\pi^2\right)$ cm/s²

$\Rightarrow T = 8$ sec $x = A\sin\omega t$

$A = 1$ cm

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$$

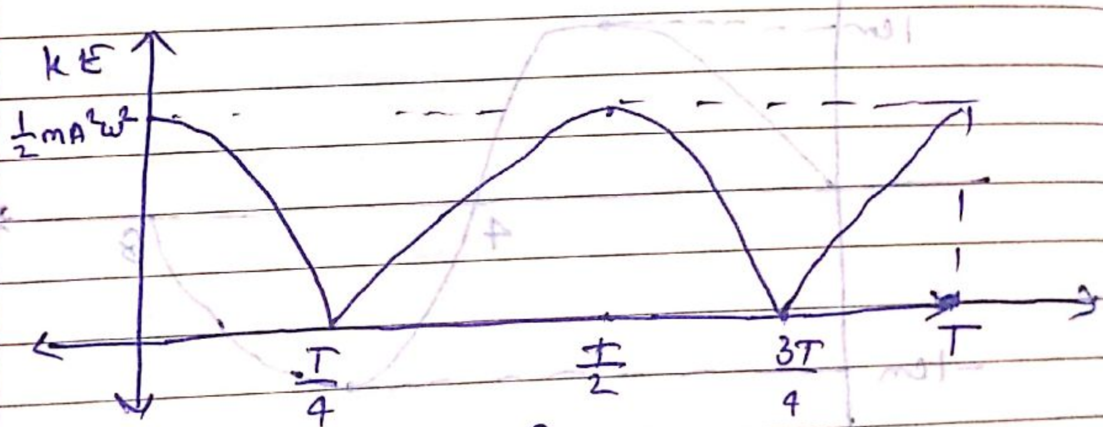
$$a = -A\omega^2 \sin\omega t$$

$$= -1 \times \frac{\pi^2}{16} \times \sin \frac{\pi}{\cancel{4}} \times \frac{\cancel{4}}{3}$$

$$= -\frac{\pi^2}{16} \times \frac{\sqrt{3}}{2}$$

$$a = -\frac{\sqrt{3}\pi^2}{32} \text{ cm/s}^2$$

- $K.E = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2\omega t$




$f = 2$

In SHM,

$x \rightarrow$ SHM $\quad A\sin\omega t \; (\omega)$

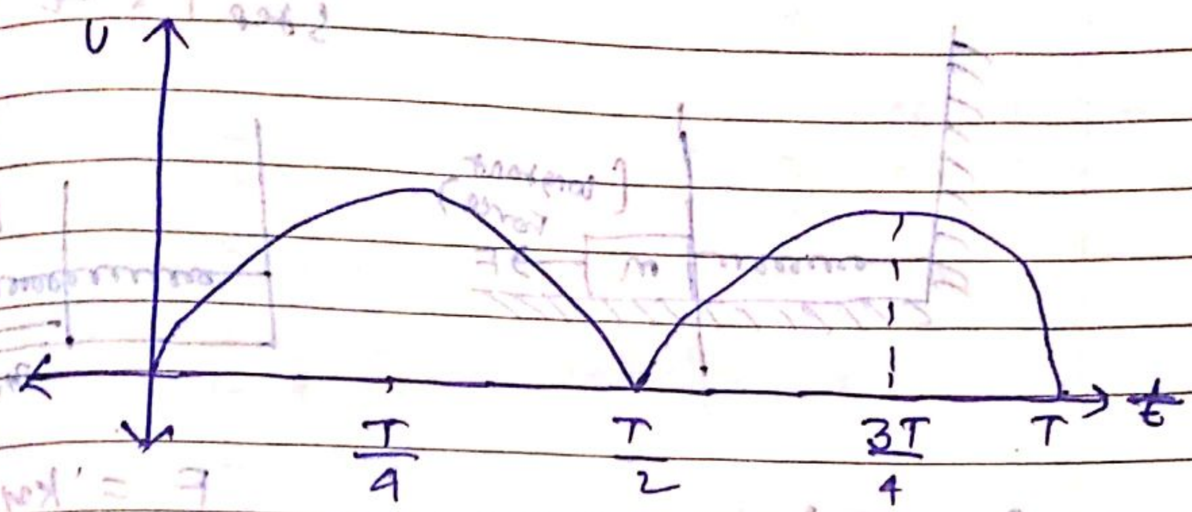
$v \rightarrow$ SHM $\quad A\omega\cos\omega t \; (\omega)$ $\quad f = 1$

$a \rightarrow$ SHM $\quad -A\omega^2\sin\omega t \; (\omega)$

K.E $\rightarrow$ ~~SHM~~ Periodic $(2\omega)$

P.E $\rightarrow$ ~~SHM~~ Periodic $(2\omega)$

$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2A^2\sin^2\omega t$




# Steps to Find Time Period of any S.H.M :-

(1) Find Equilibrium position $F_{ext} = 0$

Write Balanced force equation.

(2) Displace the object slightly (x) from mean position & find new $F_{net}$. & mathematically arrive at $F_{net} = -kx$

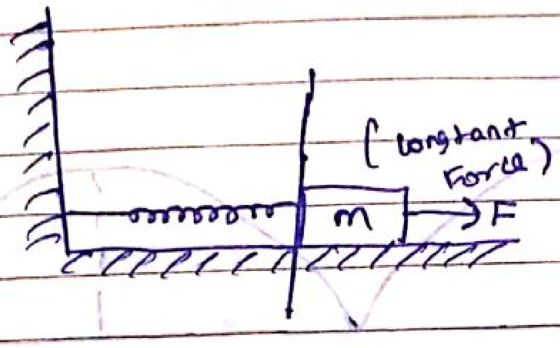
(3) $K = m\omega^2 \Rightarrow \omega = \sqrt{\frac{K}{m}}$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

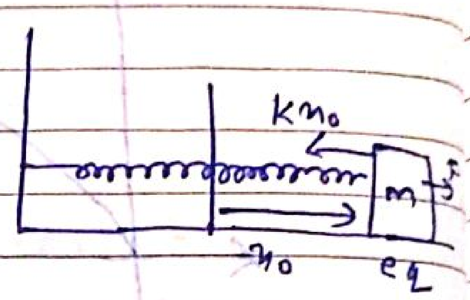
SHM constant

Spring constant only for spring block system.

• Time Period of Spring Block System :

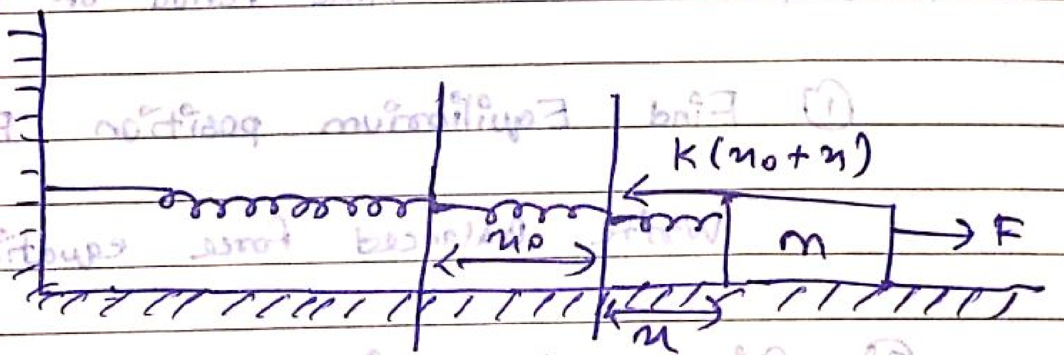



Step 1 : eq




$F = Kx_0$

Step 2 :

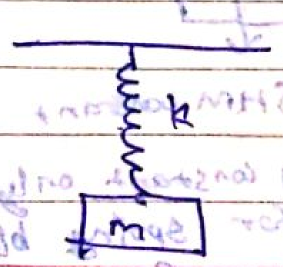



$$F_{net} = F - K(x_0 + x)$$
$$= F - Kx_0 - Kx$$

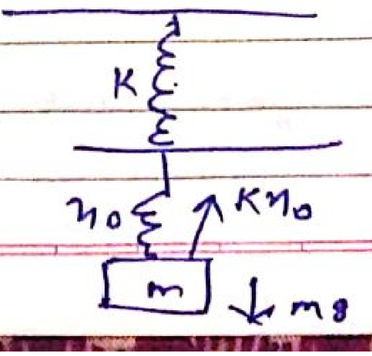
$$\boxed{F_{net} = -Kx}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

• 

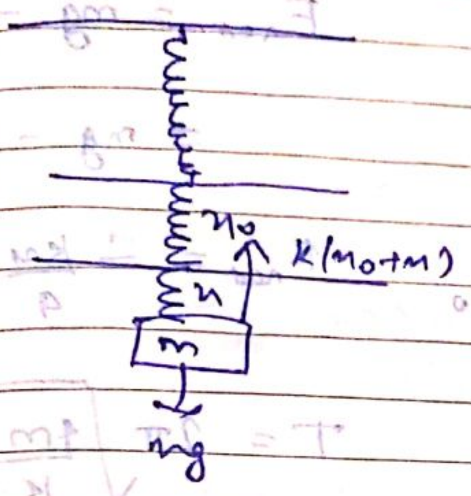



Step 1
eq
⇒



$mg = kx_0$

II :



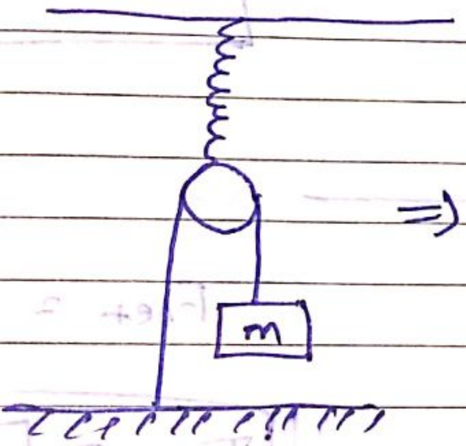

$F_{net} = mg - kx_0 - kx$

$= kx_0 - kx_0 - kx$

$\boxed{F_{net} = -kx}$

$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

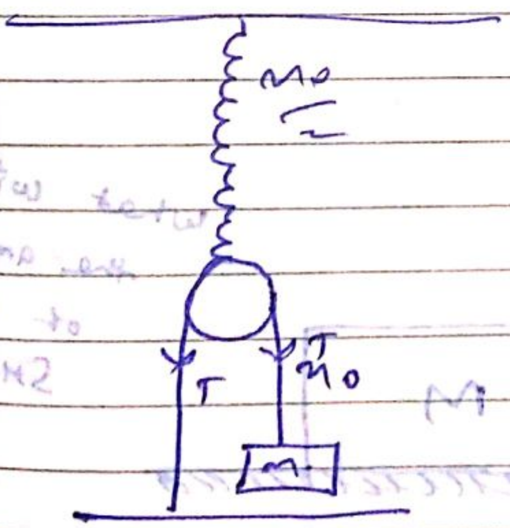
3)




$\Rightarrow$

$\cancel{F_{net} = -\frac{2kx}{2}}$

$\cancel{T = 2\pi\sqrt{\frac{2km}{2k}}}$

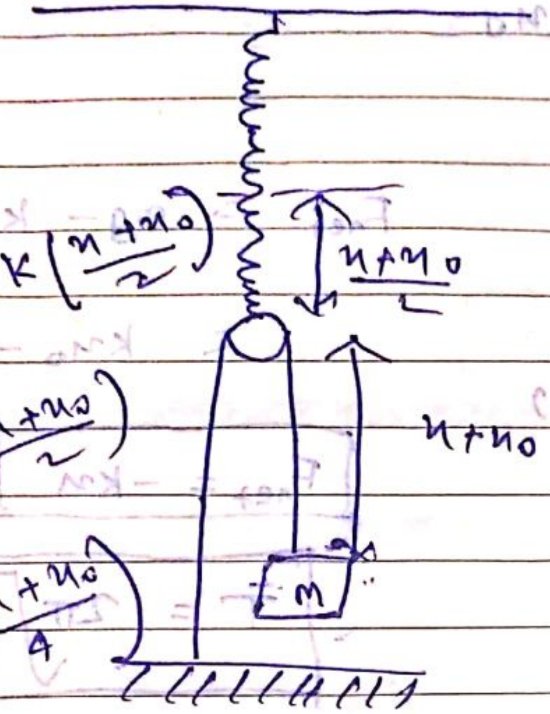



$2T = \frac{kx_0}{2}$

$T = \frac{kx_0}{4}$

$mg = T$

$mg = \frac{kx_0}{4}$




$$F_{net} = mg - T'$$

$$2T' = K\left(\frac{x+x_0}{2}\right)$$

$$= mg - \frac{Kx}{4} - \frac{Kx_0}{4}$$

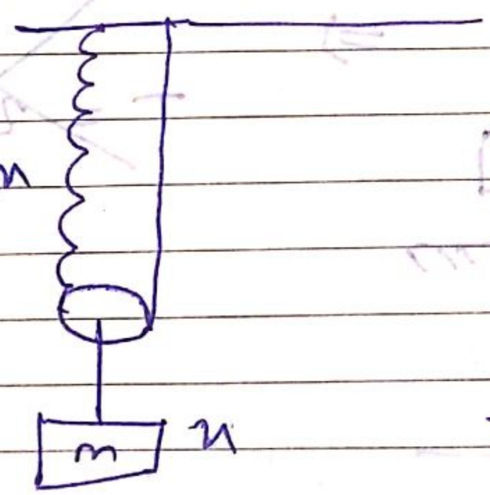
$$T' = K\left(\frac{x+x_0}{4}\right)$$

$$F_{net} = -\frac{Kx}{4}$$

$$T = 2\pi\sqrt{\frac{4m}{K}}$$

$$\boxed{T = 4\pi\sqrt{\frac{m}{K}}}$$

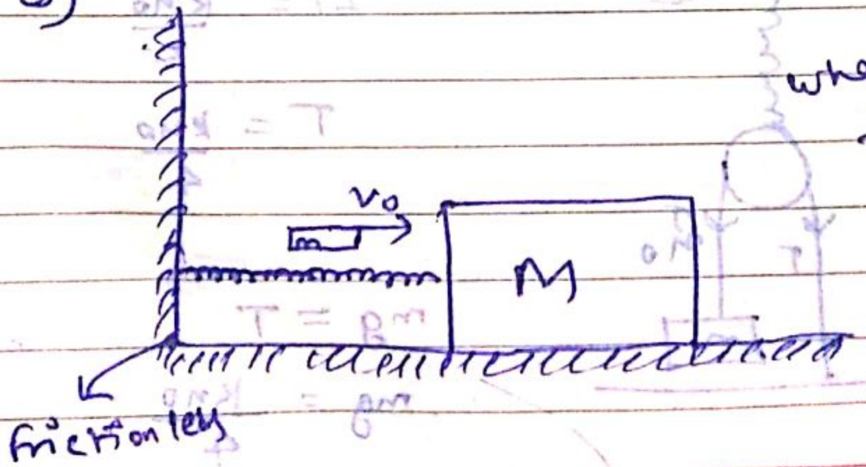
Q)




$$F_{net} = 4Kx$$

$$\boxed{T = \pi\sqrt{\frac{m}{K}}}$$

Q)




what will be the amplitude of that SHM??

SHM

$$mv_0 = (m+M)v'$$

$$v' = \frac{mv_0}{m+M}$$

$$\frac{1}{2} \times \frac{m^2 v_0^2}{m+M} = \frac{1}{2} k x^2$$

$$x^2 = \frac{m^2 v_0^2}{(m+M)k}$$

$$x = \frac{mv_0}{\sqrt{(m+M)k}}$$

$$k = (M+m)\omega^2$$

$$\omega = \sqrt{\frac{K}{m+M}} \qquad V_{mean\ max} = \omega A$$

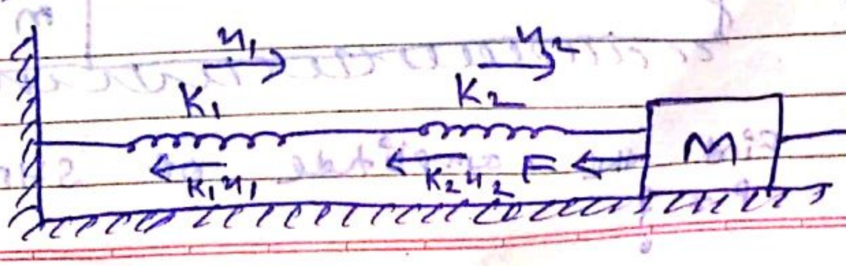
$$\frac{mv_0}{m+M} = \omega A$$

$$\frac{mv_0}{m+M} = \sqrt{\frac{k}{M+m}}\, A$$

$$\boxed{A = \frac{mv_0}{\sqrt{(M+m)k}}}$$

- Series

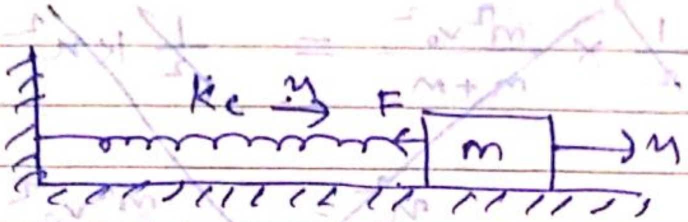
$$T = 2\pi\sqrt{\frac{m}{K_e}}$$



K ↑↑ ⟹ String stiff

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K_e = \frac{K_1 K_2}{K_1 + K_2}$$




$$F = K_e x = K_1 x_1 = K_2 x_2$$

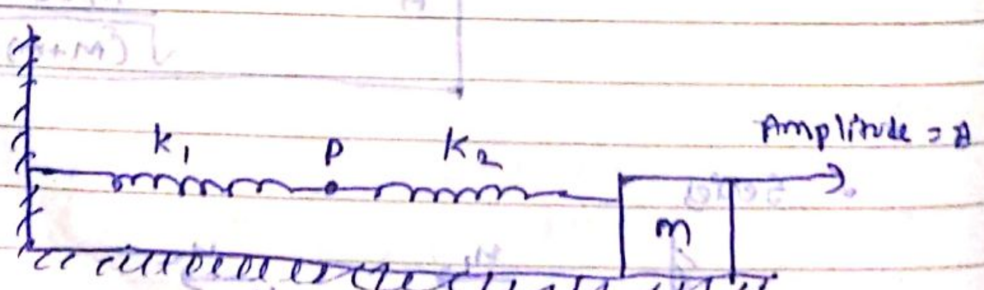
$$K_2 x_2 = F = K_1 x_1$$

$$x = x_1 + x_2$$

$$\frac{F}{K_e} = \frac{F}{K_1} + \frac{F}{K_2}$$

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

# IIT 2009




Find the amplitude of SHM of P = ?

a) $\frac{K_1 A}{K_1 + K_2}$ ~~b)~~ $\frac{K_2 A}{K_1 + K_2}$ c) $\frac{K_1 A}{K_2}$ d) $A$

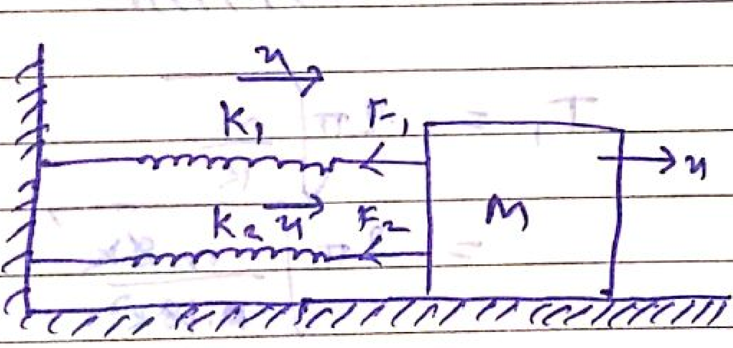
$\Rightarrow$ $\boxed{F = K_1 x_1 = K_2 x_2} = K_E A$

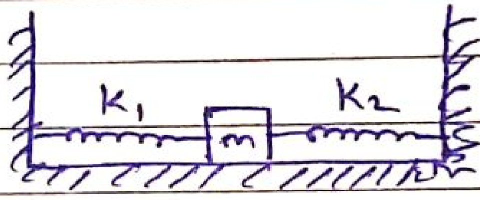
$$x_1 = \frac{K_E A}{K_1}$$

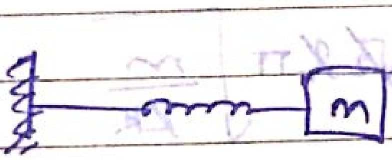
$$= \frac{K_1 K_2 A}{(K_1 + K_2) K_1}$$

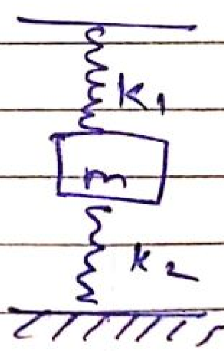
$$x_1 = \frac{K_2 A}{K_1 + K_2}$$

# Parallel

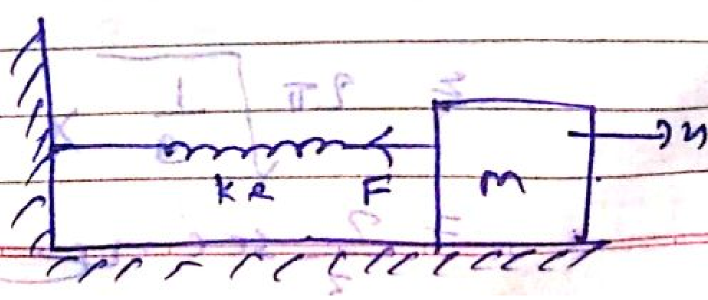



$$\boxed{K_e = K_1 + K_2}$$

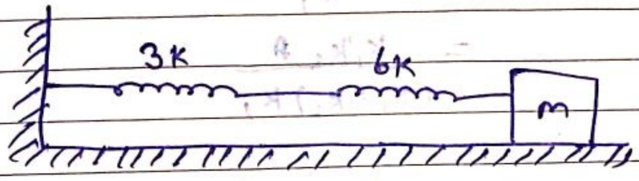


$$F = F_1 + F_2$$

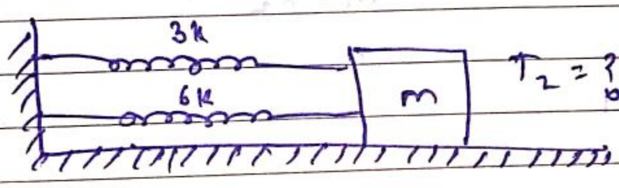
$$K_e x = k_1 x + k_2 x$$

$$\boxed{K_e = k_1 + k_2}$$

Q)




$T_1 = \sqrt{2}\,s$




$T_2 = ?$

$$T_1 = 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{m\,9K}{\cancel{18}\,K^2}}$$

$$\cancel{\sqrt{2}} = \cancel{\sqrt{2}}\,2\pi\sqrt{\frac{m}{2K}}$$

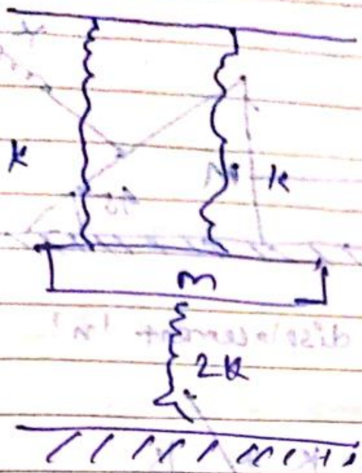
$$T_2 = 2\pi\sqrt{\frac{m}{9K}}$$

$$= 2\pi\sqrt{\frac{1}{9}} \times \frac{1}{\pi}$$

$$= \frac{2}{3}\ \text{sec}$$

Q)

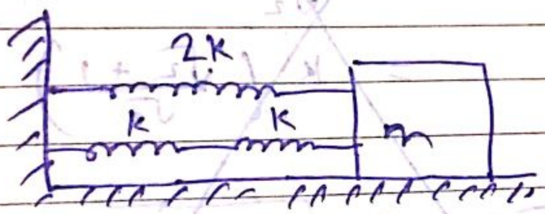
a)




$$T = 2\pi\sqrt{\frac{m}{4k}}$$

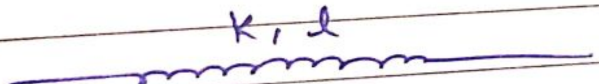
$$T_1 = \pi\sqrt{\frac{m}{k}}$$

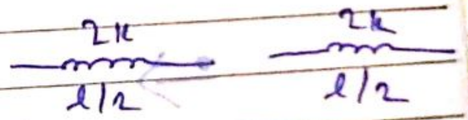
b)



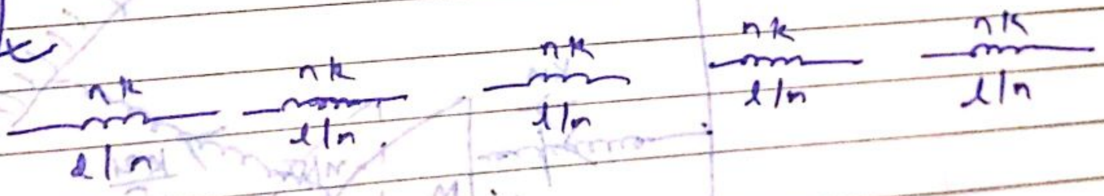

$$T_2 = 2\pi\sqrt{\frac{2m}{5k}}$$

# Cutting of Springs

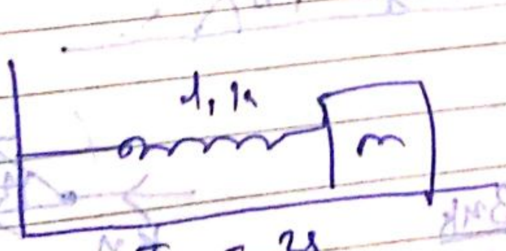


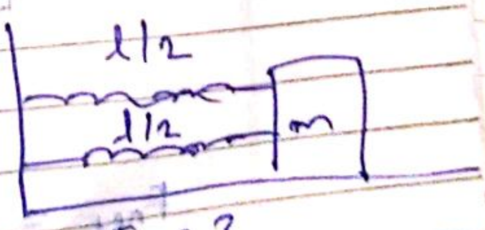

$$\boxed{K \propto \frac{1}{l}}$$




Q)



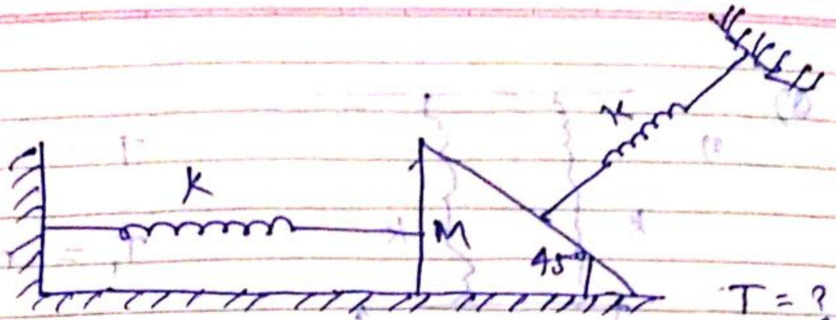

$T_1 = 2s$




$T_2 = ?$

$$\boxed{T_2 = 1\,\text{sec}}$$

Q)



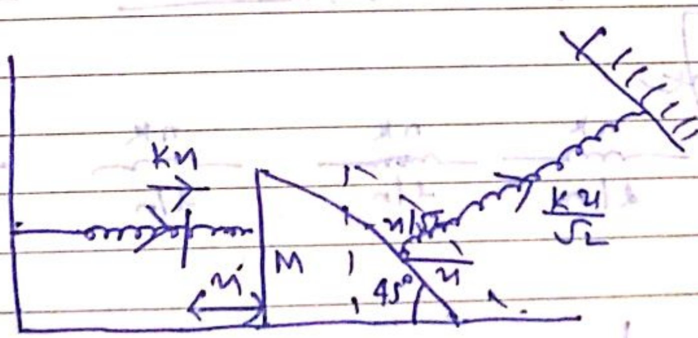

T = ?

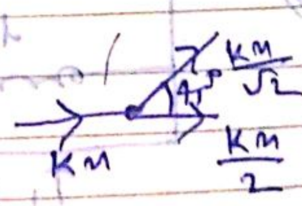
Small Horizontal displacement 'x'

$$\cancel{K_e = K + \frac{K}{\sqrt{2}} = \frac{\sqrt{2}K + K}{\sqrt{2}} = K\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)}$$

$$\cancel{T = 2\pi\sqrt{\frac{\sqrt{2}m}{(\sqrt{2}+1)K}}}$$

$\Rightarrow$

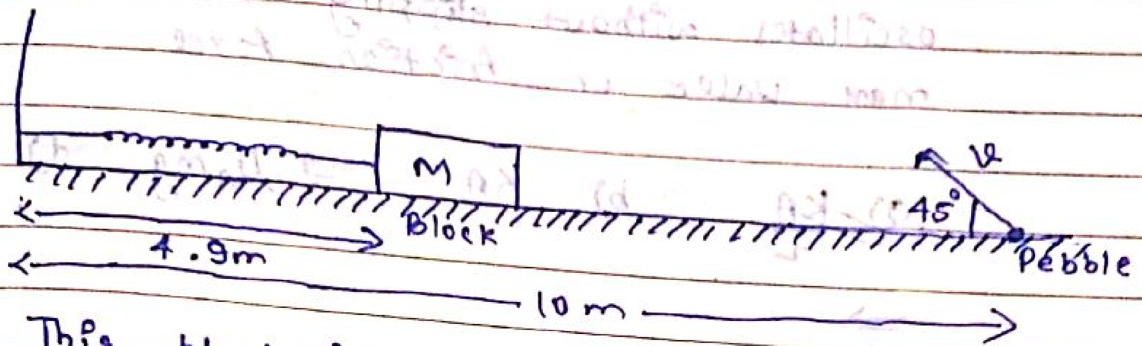



$$F_{net} = -\frac{3xk}{2}$$

$$\boxed{T = 2\pi\sqrt{\frac{m2}{3k}}}$$

# IIT 2012 :




This block is stretched to $x = 0.2$ m & released from rest at $t = 0$. Block → SHM → $\omega = \frac{\pi}{3}$. At $t = 0$ pebble is projected

If both meets at $t = 1$s, then find $v = ?$

a) $\sqrt{50}$ b) $\sqrt{51}$ c) $\sqrt{52}$ d) $\sqrt{53}$

$\Rightarrow$ $T = \frac{2\pi}{\pi} \times 3 = 6$ sec

At $t = 1$ sec,

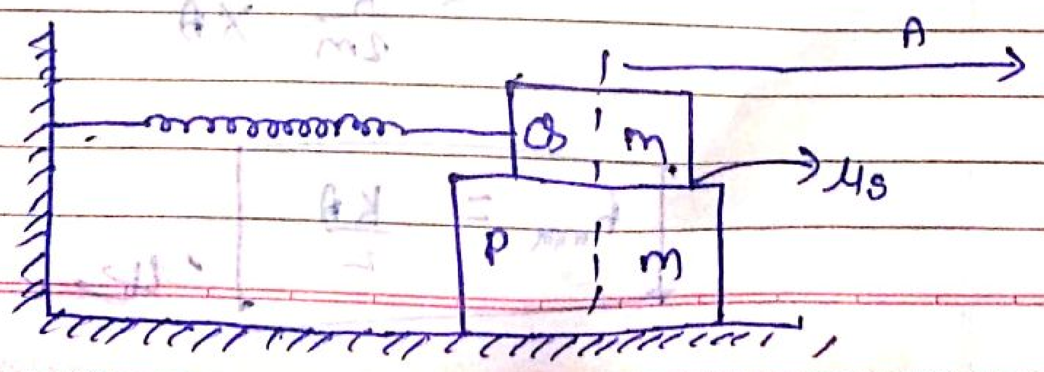
$R = 5$m

$= \frac{v^2}{g}$

$5 \times 10 = v^2$

$v = \sqrt{50}$ m/s.

# IIT 2004

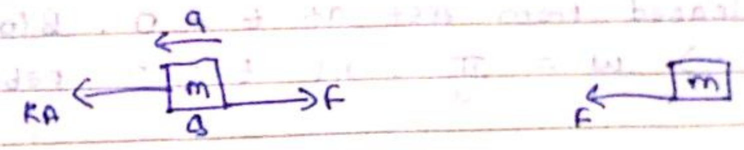


Two blocks pulled by distance A & oscillates without slipping what is the max value of friction force.

a) $\frac{KA}{2}$ b) $KA$ c) $\mu_s mg$ d) zero

$\Rightarrow$

At extreme points,

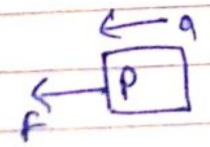



$-f + KA = ma$ $\qquad$ $f = ma$

$2f = KA$

$f = \frac{KA}{2}$ . Ans

<u>OR</u>




$f = ma$

$f_{max} = m a_{max}$

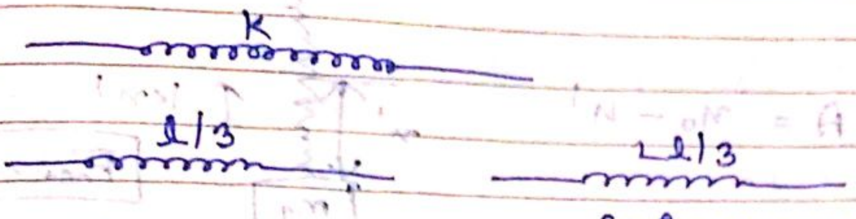
SHM,

$a_{max} = \omega^2 A$

$= \frac{K}{2m} \times A$

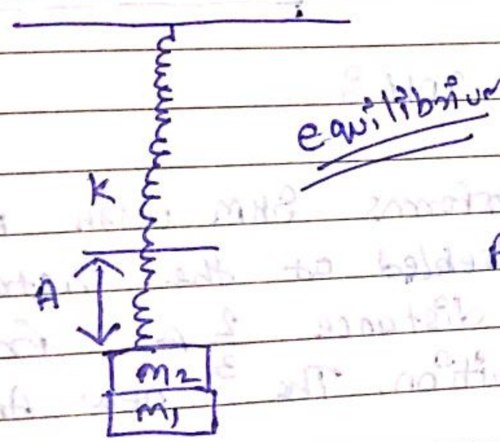
$\boxed{f_{max} = \frac{KA}{2}}$ . Ans

# IIT 1999




$\Rightarrow \quad k' = \frac{3k}{2}$

# IIT 1981 :




$m_1$ is removed without any disturbance.
$m_2$ oscillates with freq $\omega$ & Amplitude A.
$\omega = ?$ , $A = ?$

$\Rightarrow \quad K x_0 = (m_2 + m_1) g$

$$x_0 = \frac{(m_1 + m_2) g}{K}$$

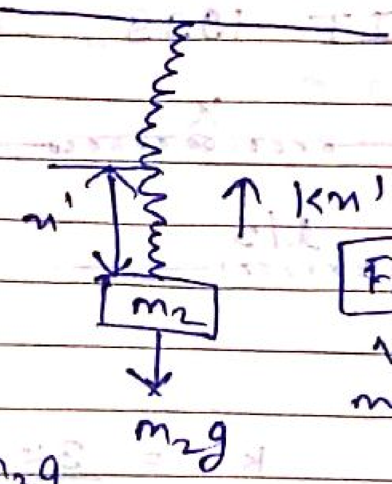
$$\cancel{KA - m_2 g = m_2 a}$$

$$\cancel{m_1 g = m_2 a}$$

$$\cancel{a = \frac{m_1 g}{m_2}}$$

$\omega =$

$A = x_0 - x'$




$\boxed{F_{net} = 0}$

v = max
mean position

$$A = \frac{m_1 g + m_2 g}{k} - \frac{m_2 g}{k}$$

$$\boxed{A = \frac{m_1 g}{k}} \qquad \boxed{\omega = \sqrt{\frac{k}{m_2}}}$$

# JEE Mains 2016 :

A particle performs SHM with Amp 'A'. Its speed is trebled at the instant when it is at a distance $\frac{2}{3}A$ from equilibrium position. The new Amplitude of SHM ⟶

a) $\frac{A\sqrt{41}}{3}$  b) $3A$  c) $A\sqrt{3}$  d) $\frac{7A}{3}$

$\Rightarrow$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\frac{1}{2} \times m \times 9 \times \omega^2 \left(\frac{5A^2}{9}\right) = \frac{1}{2}kx^2$$

$$\frac{5A^2}{2} = x^2$$

$$v_F = 3v_P$$

$$\cancel{\omega}\sqrt{A_F^2 - \left(\frac{2}{3}A\right)^2} = 3\cancel{\omega}\sqrt{A^2 - \left(\frac{2}{3}\right)A^2}$$

$$A_F^2 - \frac{4A^2}{9} = \frac{\cancel{9} \times 5A^2}{\cancel{9}}$$

$$A_F^2 = 5A^2 + \frac{4}{9}A^2$$

$$\boxed{A_F = \frac{7A}{3}}$$

# IIT 2010

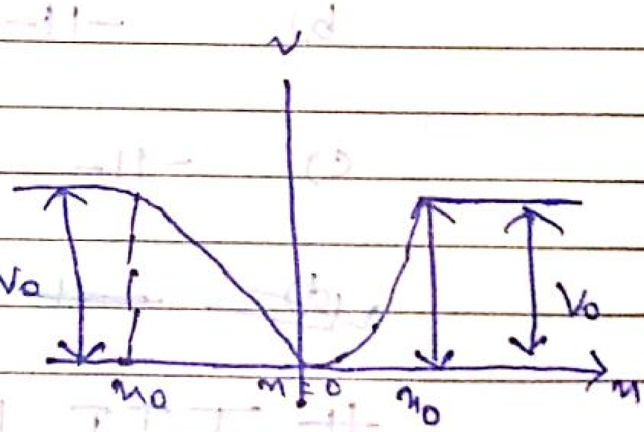
$V(x) = kx^2$ SHM, time period $\propto \sqrt{\frac{m}{k}}$

$V(x) = \alpha x^4$ dimensional analysis

$\downarrow$

increases on both sides of $x = 0$

Periodic motion




$V(x) = \alpha x^4$ $|x| < X_0$

$V(x) = V_0$ (constant) $|x| > X_0$

Total Energy $\longrightarrow$ infinity enscape X

1) If the motion is periodic & Total Energy of particle is E, then

a) $E < 0$, ~~b)~~ $E > 0$, ~~c)~~ $V_0 > E > 0$

d) $E > V_0$

② Periodic, time period proportional $\alpha$

a) $A\sqrt{\frac{m}{\alpha}}$ ~~b)~~ $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$, c) $A\sqrt{\frac{\alpha}{m}}$

d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

③ acceleration of this particle for

$|x| > X_0$

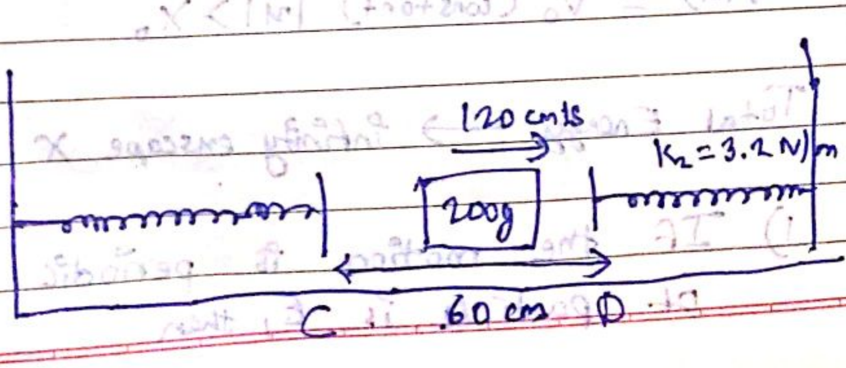
a) proportional to $V_0$

b) —||— $\frac{V_0}{m\cancel{X_0}}$

c) —||— $\sqrt{\frac{V_0}{mX_0}}$

~~d)~~ ~~—||—~~ zero

# IIT 1985



Date
Page

if mass 200g moves with 120 cm/sec between C & D. Find the time period of oscillation of 100 g mass.

a) 2.42 s  b) 2.82 s  c) 3.42 s  d) 3.62 s

$\Rightarrow$

$0.2 v^2 = 3.2 \times x^2$

$\cancel{0.2} \times 0.12 \times 0.12 = \cancel{3.2}^{16} \times x^2$

$\frac{0.12}{4} = x$

$x = 0.03$ m

$= 3\,$ cm

$t = \frac{1}{2} \times \frac{0.2 \times 0.12 \times 0.12}{0.03 \times 3.2}$

$= \frac{\cancel{2} \times \cancel{12} \times \cancel{0.12}}{\cancel{2} \times \cancel{3} \times \cancel{32}}$ 0.03

$= 0.015$ sec

2.03

$\cancel{0.2} \times 0.12 \times 0.12 = \cancel{1.8}^{9} \times x^2$

$\frac{0.12}{3} = x$

$x = 0.04$ m

$= 4\,$ cm

$t = \frac{1}{\cancel{2}} \times \frac{\cancel{0.2} \times 0.12 \times 0.12}{0.04 \times 1.8}$

$= \frac{0.12 \times \cancel{12}}{4 \times \cancel{18}\; 6}$

$= 0.02$

$T = \frac{120}{120} + \pi\sqrt{\frac{0.2}{3.2}} + \pi\sqrt{\frac{0.2}{1.8}}$

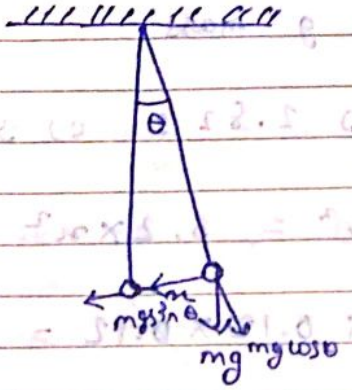
$= 1 + \pi/4 + \pi/3$

$= 1 + 0.78 + 1.04$

$= 2.82$ Sec.

Scanned by CamScanner

# Time Period of Simple Pendulum :




if θ is small, then path is a straight line ⟹ SHM

$$\theta = \frac{x}{l}$$

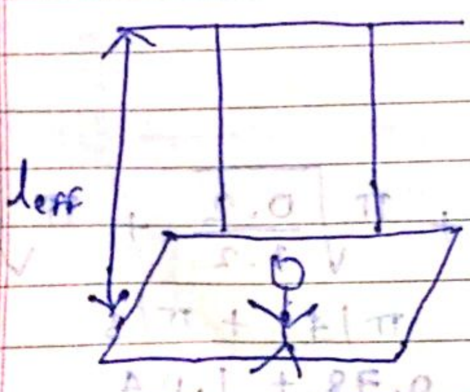
$$\sin\theta \approx \theta \approx \frac{x}{l}$$

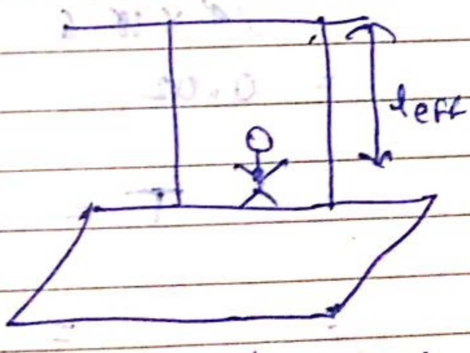
$$F = -mg\sin\theta$$
$$= -\frac{mgx}{l}$$
$$-kx = -\frac{mgx}{l}$$

$$k = \frac{mg}{l}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$= 2\pi\sqrt{\frac{ml}{mg}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$



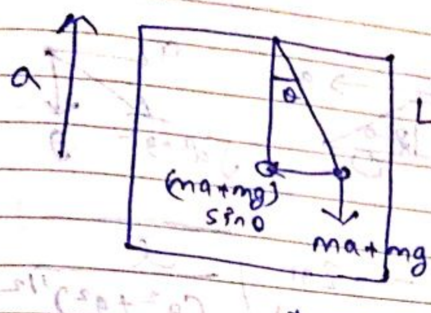




$$T = 2\pi\sqrt{\frac{l_{eff}}{g}}$$

$l_{eff} \downarrow$ $T \downarrow$

$\omega \uparrow$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g_{eff}}}}$$




$F = -\frac{(ma+mg)x}{l}$

$k = \frac{ma+mg}{l}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{a+g}}}$$

# IIT JEE 2005

Pendulum $T_1$

Point of suspension move upward

$y = kt^2$ ($k = 1 m/s^2$) ($t \rightarrow$ time)

new Time Period $\longrightarrow T_2$

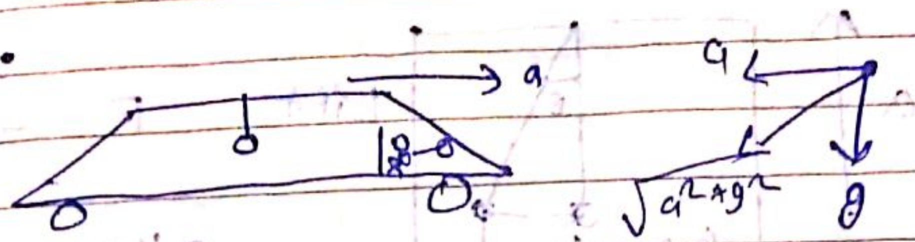
$\frac{T_1^2}{T_2^2} = ?$

a) $\frac{5}{6}$ b) $\frac{6}{5}$ ✓ c) 1 d) $\frac{1}{2}$

$\Rightarrow$ $y = kt^2$

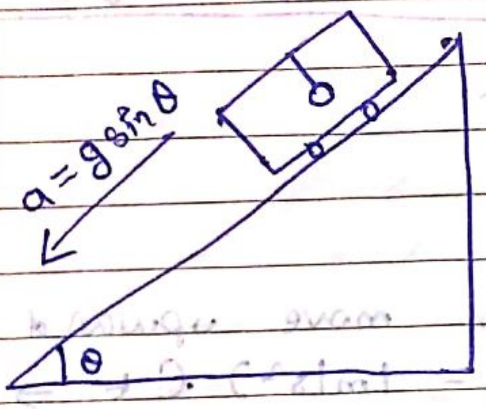
$a = 2$ sec

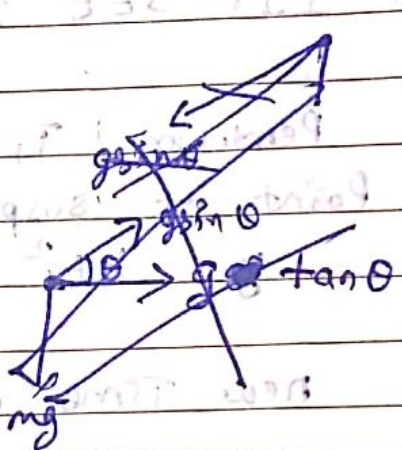
$\frac{T_1^2}{T_2^2} = \frac{12}{10} = \frac{6}{5}$

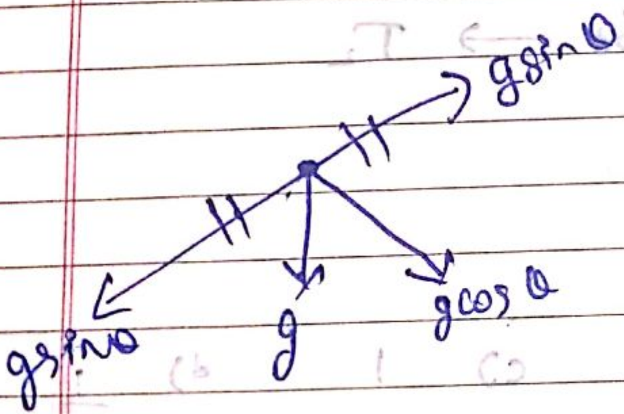



$$T = 2\pi\sqrt{\frac{l}{(a^2+g^2)^{1/2}}}$$

$T = ?$




$$\boxed{T = 2\pi\sqrt{\frac{l}{g\cos\theta}}}$$

## * Physical Pendulum :

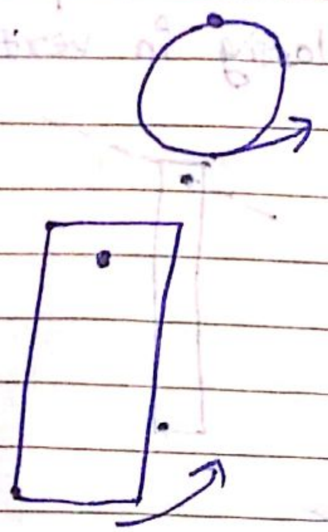
| Linear SHM | Angular SHM |
| --- | --- |
| ① $F = -kx$ | $\tau = -k\theta$ |
| ② $T = \frac{2\pi}{\omega}$ | $T = \frac{2\pi}{\omega}$ |

Date ____
Page ____

③ $\omega = \sqrt{\frac{K}{m}}$ ; $\omega = \sqrt{\frac{K}{I}}$

④ $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$ ④ $\boxed{T = 2\pi\sqrt{\frac{I}{k}}}$

---

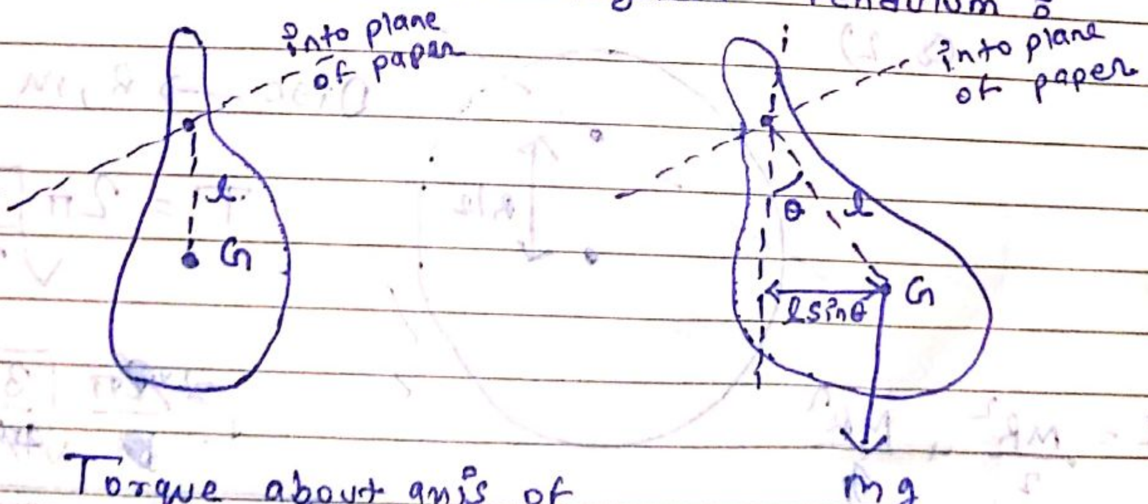


Rigid Body (pivoted) oscillate

of Any shape

oscillations should be very very small

- Time Period of Physical Pendulum :




Torque about axis of rotation :

$$\tau = mg(l\sin\theta)$$
$$= -mgl\theta$$

$$\tau = -K\theta$$

$\therefore \boxed{K = mgl}$

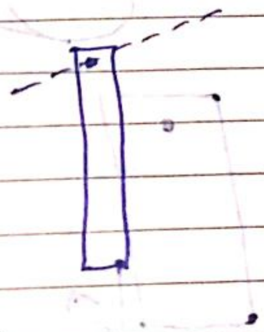
$$T = 2\pi\sqrt{\frac{I}{k}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgl}}}$$

$I \rightarrow$ about axis of rotation

Q) Rod of Length L, Mass M, pivoted about one end & oscillating in vertical plane.

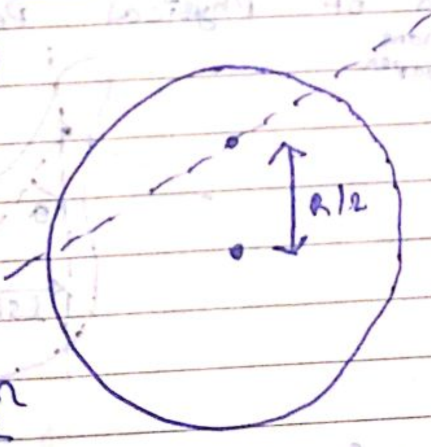
$T = ?$



$$T = 2\pi\sqrt{\frac{\cancel{m}l^{\cancel{2}}}{3\cancel{m}g\cancel{l}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{3g}}}$$

Q.2)




Disc $\rightarrow$ R, M

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

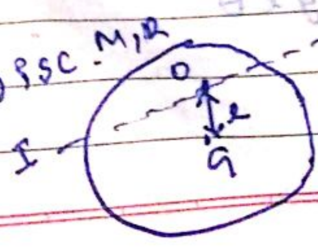
$$= 2\pi\sqrt{\frac{3\cancel{m}l^{\cancel{2}} \times 2}{4\cancel{m}g\cancel{l}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{3l}{2g}}}$$

$$I = \frac{MR^2}{2} + \frac{MR^2}{4}$$

$$= \frac{3MR^2}{4}$$

Q) Disc M, R



For what distance OG.

① Time Period $\longrightarrow$ Max

② Time Period $\longrightarrow$ Min

Also find these Two Time Periods.

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{mgl}}$$

$$T = 2\pi\sqrt{\frac{\frac{MR^2}{2} + ML^2}{MgL}}$$

$$y = \frac{R^2}{2gL} + \frac{L}{g}$$

$T \rightarrow$ min, $y \rightarrow$ min

$$\frac{dy}{dL} = 0$$

$$\frac{-R^2}{2gL^2} + \frac{1}{g} = 0$$

$$L^2 = \frac{R^2}{2}$$

$$L = \pm\frac{R}{\sqrt{2}}$$

$$T = 2\pi\sqrt{\frac{\left(\frac{R^2}{2} + \frac{R^2}{2}\right)\sqrt{2}}{Rg}}$$

$$= 2\pi\sqrt{\frac{\sqrt{2}R}{g}}$$

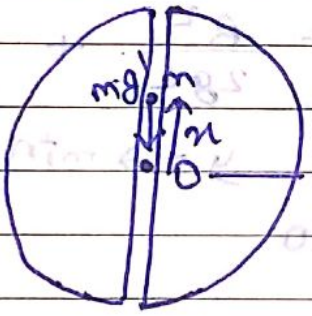
$$\boxed{T = 2\pi\sqrt{\frac{1.414R}{g}}}$$

# $T_{max} \longrightarrow \infty$

$$\boxed{l = 0}$$

- Tunnel through Earth :

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}} \approx 84.6 \text{ min}$$




Mean position $\{F_{net} = 0,\ g = 0\}$

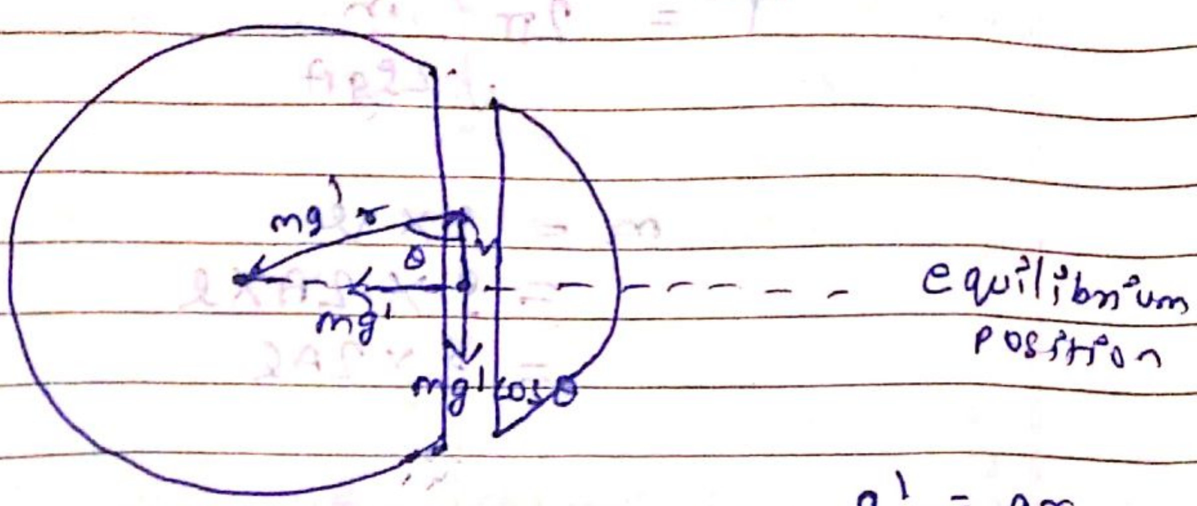
$$F_{net} = mg' = -\frac{mgx}{R}$$

$$k = \frac{mg}{R}$$

$$T = 2\pi\sqrt{\frac{mR}{mg}}$$

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$$

$\approx 84.6$ min.




$$g' = \frac{gr}{R}$$

$$F_{net} = \frac{mgr}{R}\cos\theta$$

$$= \frac{mg\cancel{r}}{R} \times \frac{x}{\cancel{r}}$$

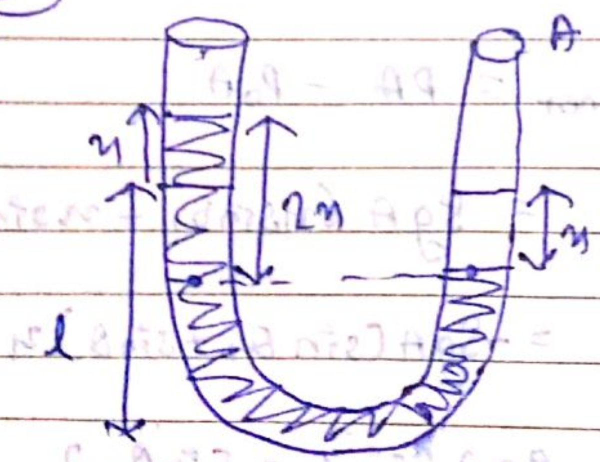
$$= -\frac{mgx}{R} \qquad k = \frac{mg}{R}$$

$$-k\cancel{x} = -\frac{mg\cancel{x}}{R}$$

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$$

- Oscillation of Liquid in a U-tube :

ρ




$$F_{net} = PA - P_0A$$

$$F_{net} = (P_0 + \rho g 2x)A - P_0A$$

$$= -\rho g 2xA$$

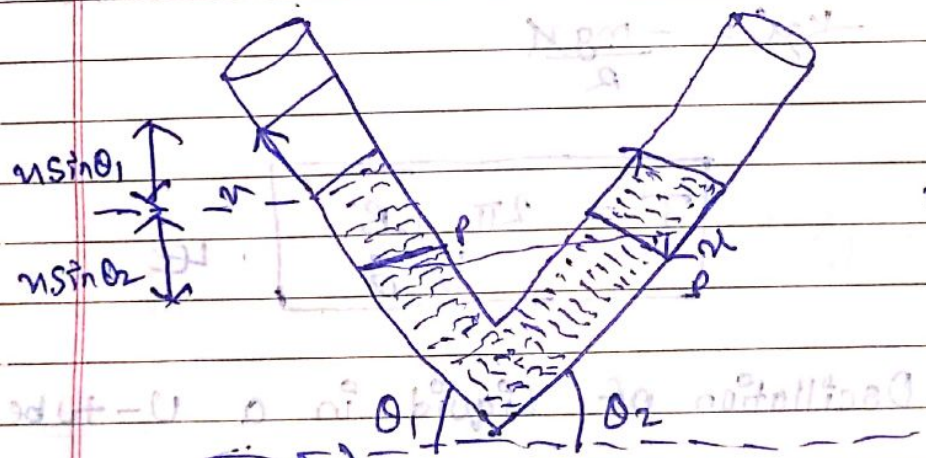
$$F_{net} = -2\rho g A x$$

$$T = 2\pi\sqrt{\frac{m}{2\rho g A}}$$

$$m = \rho \times V$$
$$= \rho \times 2A \times l$$
$$= \rho \times 2Al$$

$$T = 2\pi\sqrt{\frac{\cancel{2}\cancel{\rho}\cancel{A}l}{\cancel{2}\cancel{\rho}g\cancel{A}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}} \quad 4$$



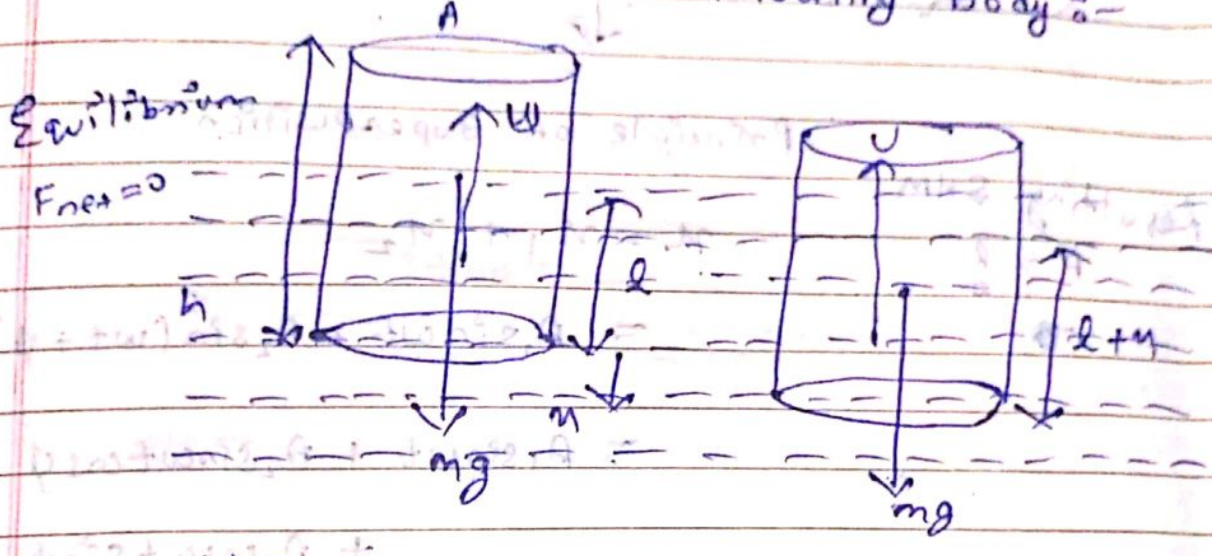

$m, A, \rho$

$P = P_0 + \rho g(x\sin\theta_1 + x\sin\theta_2)$

$$F_{net} = PA - P_0A$$
$$= \rho g A(x\sin\theta_1 + x\sin\theta_2)$$
$$= -\rho g A(\sin\theta_1 + \sin\theta)x$$
$$K = \rho g A(\sin\theta_1 + \sin\theta_2)$$

$$T = 2\pi\sqrt{\frac{m}{\rho g A(\sin\theta_1 + \sin\theta_2)}}$$

• Oscillation of a Floating Body :-




$U = mg$

$V\rho g = mg$

$A l \rho g = mg$ — (1)

$F_{net} = U - mg$

$= V'\rho g - mg$

$= A(l+x)\rho g - mg$

$= A l \rho g + A x \rho g - mg$

$F_{net} = -A\rho g x$

$F_{net} = -kx$

$k = A\rho g$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

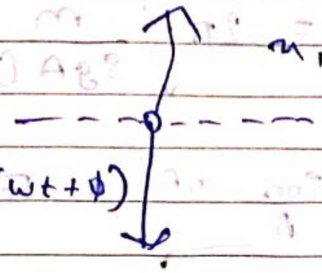
$$= 2\pi\sqrt{\frac{A l \rho}{A\rho g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Date
Page

• Combination of SHM :-

$x_1 = A_1 \sin \omega t$



$x_2 = A_2 \sin(\omega t + \phi)$

Principle of Superposition

Resulting SHM

$A = ?$

$$x = x_1 + x_2$$

$$= A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$$

$$= A_1 \sin \omega t + A_2 \sin \omega t \cos \phi + A_2 \cos \omega t \sin \phi$$

$$= \sin \omega t \, \underbrace{(A_1 + A_2 \cos \phi)}_{A \cos \theta} + \cos \omega t \, \underbrace{(A_2 \sin \theta)}_{A \sin \theta}$$

$$x = A \sin \omega t \cos \theta + A \cos \omega t \sin \theta$$

$$\boxed{x = A \sin(\omega t + \theta)}$$

$$A \sin \theta = A_2 \sin \phi \quad \text{—— (I)}$$

$$A \cos \theta = A_1 + A_2 \cos \phi \quad \text{—— (II)}$$

Date ______
Page ______

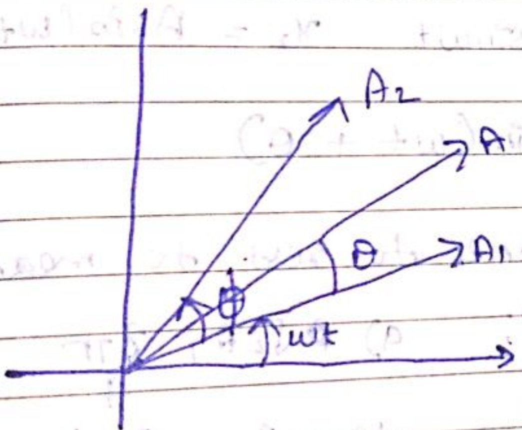
$$A^2 = A_2^2 \sin^2\phi + A_1^2 + A_2^2\cos^2\phi + 2A_1A_2\cos\phi$$

$$\boxed{A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Divide (i) ÷ (ii)

$$\boxed{\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}}$$

## # Shortcuts :-




$x_1 = A_1 \sin\omega t$

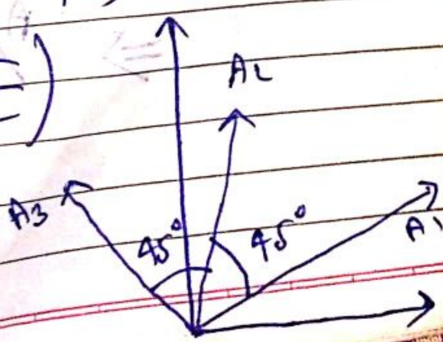
$x_2 = A_2 \sin(\omega t + \phi)$

$x = A\sin(\omega t + \theta)$

(Q) $x_1 = 2\sin\omega t$

$x_2 = 2\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$

$x_3 = \sin\left(\omega t + \frac{\pi}{2}\right)$




$\Rightarrow A_x = 2 + 2 = 4$

$A_y = 1 + 2 = 3$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$= 5$$

$$\tan\theta = \cancel{\frac{4}{3}}\ \frac{3}{4}$$

$$\boxed{\theta = 37^\circ}$$

$$\boxed{x = 5\sin(\omega t + 37^\circ)}$$

Q) IIT 2011

$x_1 = A\sin\omega t \qquad x_2 = A\sin\left(\omega t + \frac{2\pi}{3}\right)$

$x_3 = B\sin(\omega t + \theta)$

particles comes to rest to mean position.

B, $\theta$ = ?

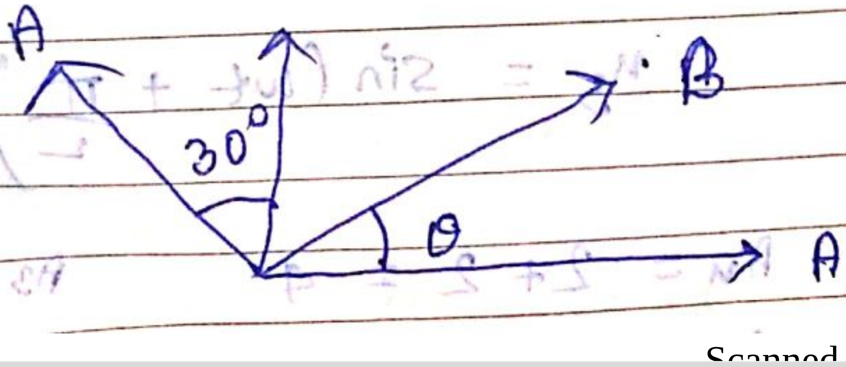
a) $\sqrt{2}A, \frac{3\pi}{4}$

~~b)~~ $A, 4\pi/3$

c) $\sqrt{3}A, \frac{5\pi}{6}$

~~d)~~ $A, \pi/3$

$\Rightarrow$



Date______
Page______

$$A + B\cos\theta - \frac{A}{2} = 0$$

$$B\cos\theta = -\frac{A}{2}$$

$$\frac{\sqrt{3}A}{2} + B\sin\theta = 0$$

$$B\sin\theta = -\frac{\sqrt{3}A}{2}$$

$$\tan\theta = \sqrt{3}$$

$\boxed{\theta = 60^\circ}$ But, ∵ B is opp.

$\boxed{\theta = 240^\circ}$ .ue

$\boxed{B = A}$ .ue